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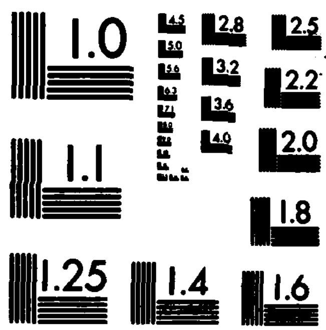

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MELBOURNE, VICTORIA

Aerodynamics Technical Memorandum 365

ESTIMATION OF HELICOPTER PERFORMANCE USING A PROGRAM BASED ON BLADE ELEMENT ANALYSIS.

by

A. M. ARNEY

Approved for Public Release

JULY 1984

COPY No

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ESTIMATION OF HELICOPTER PERFORMANCE USING A PROGRAM BASED ON BLADE ELEMENT ANALYSIS

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SUMMARY

A convenient method of predicting helicopter performance is presented, which is applicable up to speeds corresponding to an advance ratio of 0.3, for any conventional helicopter (i.e. single main rotor) with flapping blades. This method uses a computer program, 'POLAR', which is based on a blade-element analysis assuming uniform induced flow. Program 'POLAR' can be employed for most steady flight conditions and is not subject to limitations imposed by the use of performance tables and charts. Details of blade operating conditions may be estimated at specified points on the rotor disc. The structure of the program and examples of its use are given. Comparisons of estimates obtained using 'POLAR', with other performance methods are also included.

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CONTENTS (CONT.)

NOTATION

B	tip loss factor; blade elements outboard of radius BR are assumed to have profile drag but no lift.
C_{P_o}	profile-drag power coefficient, $P_o/[\rho\pi R^2(\Omega R)^3]$
C_T	thrust coefficient, $T/[\rho\pi R^2(\Omega R)^2]$
D_p	parasite drag of helicopter fuselage
E	endurance
I	mass moment of inertia of one blade about a flapping hinge (1/3 mR^2)
L	lift
$M_{(x,\psi)}$	Mach number at radial position x, azimuth ψ
P_{avail}	power available at main rotor shaft
P_c	climb power, i.e. excess power available for climb
P_{comp}	power component arising from compressibility effects on advancing blade
P_i	power component arising from rotor lift (Induced Power)
P_{inst}	installed power i.e. total power output of helicopter engines
P_o	power component arising from rotor drag (Profile Power)
P_p	power component arising from parasite drag
P_{req}	power required at the main rotor shaft
P_{sum}	$P_i + P_o + P_p$
R	blade tip radius
SFC	specific fuel consumption
T	main rotor thrust
T_∞	main rotor thrust out of ground effect
U	resultant velocity perpendicular to blade-span axis at blade element
U_p	component at blade element of resultant velocity perpendicular both to blade-span axis and U_T

NOTATION (CONT.D)

U_T	component at blade element of resultant velocity perpendicular to blade-span axis and to ANF
V	airspeed of helicopter along flight path
ΔV	airspeed increment
V_{max}	maximum value of V
V_v	rate of climb
$\bar{V}$	V/ν_{hov}
W	all up weight of helicopter
W_f	weight of fuel
a	slope of curve of section lift-coefficient against section angle of attack
a_o	constant term in Fourier series that expresses β hence, the rotor coning angle
a_1	coefficient of $\cos\psi$ in Fourier series that expresses β hence, first-order longitudinal flapping angle
a_s	speed of sound
b	number of main rotor blades
b_1	coefficient of $\sin\psi$ in Fourier series that expresses β hence, first order lateral flapping angle
c	blade section cord
c_e	equivalent blade chord, $(\int_o^R cr^2\,dr)/(\int_o^R r^2\,dr)$
c_l	blade section lift coefficient
$\bar{c}_l$	average section lift coefficient in reversed-velocity region
c_{d_o}	blade section profile-drag coefficient
$\bar{c}_{d_o}$	average section profile-drag coefficient in reversed velocity region
f	equivalent flat-plate area of helicopter
k	semi-empirical factor for induced velocity calculation (Eq.9)
m	mass of single rotor blade
r	blade element radius

NOTATION (CONT.D)

u	non-dimensional resultant velocity at blade element, $U/\Omega R$
u_p	non-dimensional component of resultant velocity at blade element, $U_p/\Omega R$
u_T	non-dimensional component of resultant velocity at blade element, $U_T/\Omega R$
x	ratio of blade element radius to blade tip radius, i.e. r/R
z	height of rotor above ground level
Δ	$\rho/\rho_{ISA+15°C}$
Ω	angular velocity of rotor
α	angle of attack of rotor, i.e. angle between axis of no feathering and plane perpendicular to flight path. (Positive when axis is pointing rearward).
α_r	angle of attack of blade-element, measured from line of zero lift
$\alpha_{(x,\ \psi)}$	α_r at radial position x, azimuth ψ
β	blade flapping angle at particular azimuth position, measured from plane perpendicular to ANF
γ	Lock's number, $c\rho a\ R^4/I$
γ_c	climb angle, i.e. angle between flight path and horizontal (positive when climbing)
$\delta_0, \delta_1, \delta_2$	coefficients in power series expressing c_{d_o} as a function of α_r ($c_{d_o} = \delta_0 + \delta_1\alpha_r + \delta_2\alpha_r^2$)
θ	blade section pitch angle, i.e. angle between zero lift line of blade section and plane perpendicular to the ANF
θ_o	blade pitch angle at hub
θ_{tip}	blade pitch angle at tip
θ_1	blade twist, i.e. $\theta_{TIP} - \theta_o$
λ	inflow ratio, $(V\sin\alpha - \nu)/(\Omega R)$
μ	tip speed or advance ratio, $(V\cos\alpha)/(\Omega R)$
ν	induced flow velocity at rotor (always positive)

NOTATION (CONT.D)

ν_{hov}	momentum theory value of ν at hover, $\Omega R\sqrt{C_T/2}$
$\bar{\nu}$	ν/ν_{HOV}
ν_f	factored induced velocity i.e. ν multiplied by a semi-empirical factor to allow for non-uniform distribution of ν over the rotor disc
ρ	density of air
$\rho_{SL_{ISA+15°C}}$	density of air at sea-level, ISA+15°C conditions
σ	blade solidity, i.e. the ratio between blade area and disc area, $bc_e/\pi R$
ϕ	inflow angle at blade element in plane perpendicular to blade-span axis
ψ	blade azimuth angle measured in direction of rotation (zero at downwind position)

GLOSSARY

ANF	Axis of No-Feathering; defined as the axis about which there is no cyclic pitch change, i.e. an observer rotating on this axis would observe blade flapping but no pitch change.
ARDU	Aircraft Research and Development Unit.
ICAO	International Civil Aviation Organization.
IGE	In Ground Effect.
ISA	International Standard Atmosphere.
MCEP	Maneuver Criteria Evaluation Program (Reference 3 and 4).
NFP	No Feathering Plane; plane perpendicular to ANF.
OGE	Out of Ground Effect.
QNH	Equivalent sea-level ambient pressure determined from ambient pressure at airfield and airfield altitude.
RAN	Royal Australian Navy.
S.I.	Systeme Internationale (International System of units).
SFC	Specific Fuel Consumption (applied to helicopter engines in this case).
TPP	Tip Path Plane; defined as the plane which is traced out by the blade tips of the rotor, i.e. an observer on this plane would observe blades changing pitch but not flapping.

NOTE ON UNIT SYSTEMS

The programs described are capable of processing data in both S.I. and Imperial systems. The latter is the major system of this report since aeronautical practice still makes use of the Imperial system. The programs always input and output airspeed in knots and power in horsepower regardless of which system is used.

1. INTRODUCTION

In early 1983, the Aircraft Behaviour Studies - Rotary Wing Group at ARL, was asked to supply performance estimates for various helicopters being considered for purchase by the RAN. These estimates included power required in steady level and steady climbing flight, range, endurance and ceiling approximations. Two methods of performance estimation were readily available:

a) Performance charts (Refs.1 and 2).

b) Program 'MCEP' (Maneuver Criteria Evaluation Program, Refs.3 and 4).

These performance charts, while giving an indication of the onset of blade stall, have some limitations:

(i) Each set of charts apply to one particular aerofoil section; Ref.1 uses NACA 0012, Ref.2 uses NACA 23012.

(ii) The charts are restricted to certain flight conditions, which may necessitate interpolation between a series of charts.

(iii) The charts are sometimes difficult to read accurately, with small reading errors often resulting in large discrepancies in the power components.

(iv) Many of the charts of reference 1 apply only to very high performance helicopters (high advance ratios) which, at present, do not exist.

(v) The analysis of reference 2 uses momentum theory, which assumes uniform inflow, to find ν. In practice, at low speed, the average ν over the disc is higher due to the effects of blade tip-loss and non-uniform inflow. Thus the charts of reference 2 underestimate the induced power.

(vi) For some performance calculations it is necessary to produce a speed-power polar. Using charts to create such a polar would be a slow and laborious task.

Program 'MCEP' is a large and complex program which can be used to evaluate a wide range of helicopter maneouvres. 'MCEP' can also estimate a level flight speed-power polar, but the method of outputting data is unsatisfactory for present applications. For this reason program 'ENERGY' has been produced at ARL (see Appendix 4), which uses the same relatively simple expressions for power estimation and induced velocity as 'MCEP', but is much smaller and outputs data in a more suitable form. While 'ENERGY' gives accurate results in many cases (see Ref.4), certain deficiencies are evident.

(i) The occurrence of blade stall is not recognized;

(ii) Blade twist is not taken into account.

Power variations due to these factors would therefore not be predicted by 'ENERGY', thus causing errors in performance estimation.

In order to overcome the above-mentioned deficiencies of these performance estimation methods, program 'POLAR' has been created, which uses a simplified blade-element analysis, based on the work of Gessow and Crim (Ref. 5), and is applicable to any conventional helicopter (i.e. single main rotor) with flapping blades.

Due to assumptions made and the resulting simplification of the analysis, 'POLAR' is applicable for $\mu \leq 0.3$. It should be noted that this limit is not exceeded by most conventional helicopters operational at present.

A description of the equations used and assumptions made is given in Section 2, while Section 3 gives a description of 'POLAR's program structure. Section 4 is a User's guide to 'POLAR' which demonstrates the program's use in estimating helicopter performance. A comparison between 'POLAR' and other methods of performance estimation is provided in Section 5.

2. EQUATIONS

This section presents the equations used by program 'POLAR'. The method by which these equations are solved is given in Section 3.

2.1 Forces In Steady Flight

Figure 1 shows the forces acting on the helicopter in steady powered flight. The resultant thrust of the main rotor is assumed to be perpendicular to the Tip Path Plane. Bramwell (Ref.6) shows that this assumption is accurate.

Resolving forces vertically and horizontally gives*:

$$W + Dp\ \sin\gamma_c = T\cos\left[\ (-\alpha) - \gamma_c - a_1\ \right] \qquad ..(1)$$

$$Dp\ \cos\gamma_c = T\sin\left[\ (-\alpha) - \gamma_c - a_1\right] \qquad ..(2)$$

From equation (1),

$$T = \frac{W + Dp\ \sin\gamma_c}{\cos(\alpha+\gamma_c+a_1)} \qquad ..(3)$$

where

$$Dp = \frac{1}{2}\rho V^2 f \qquad ..(4)$$

Equivalent flat plate area, f, is assumed to be independent of fuselage angle of attack.

Dividing equation (2) by equation (1), and re-arranging gives:

$$\alpha = -\left[\arctan\left(\frac{Dp\ \cos\gamma_c}{W+Dp\ \sin\gamma_c}\right) + \gamma_c + a_1\right] \qquad ..(5)$$

2.2 Induced Velocity Relationships

Initial estimates of inflow ratio λ, required by 'POLAR' (see §3.2), are calculated in the following manner.

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* In NACA notation, which is used here, α as shown in figure 1 is defined to be negative.

From momentum theory:

$$\lambda = \mu \tan \alpha - \frac{C_T}{2(\mu^2 + \lambda^2)^{\frac{1}{2}}}$$

which, for hover (i.e., μ=0) reduces to

$$\lambda = - \sqrt{C_T/2} \qquad ..(6)$$

and in forward flight, assuming α is small and $\lambda \ll \mu$, gives

$$\lambda = - C_T \;/\; (2\mu) \qquad ..(7)$$

After the initial estimate for λ is made using the above equations, ν (and then λ, see §3.2) is calculated according to the standard resultant velocity equation:

$$\bar{\nu} = (\bar{V}^2 + \bar{\nu}^2 - 2 \bar{V} \bar{\nu} \sin\alpha \quad)^{-\frac{1}{2}} \qquad ..(8)$$

where $\bar{\nu} = \nu/\nu_{hov}$, $\bar{V} = V/\nu_{hov}$ and $\nu_{hov} = \Omega R\sqrt{C_T/2}$ which can be solved iteratively, assuming $\bar{V}$ and α are known.

At low speeds however, the value of ν obtained above is not realised in practice, because the induced velocity distribution is non-uniform over the rotor disc. Below μ = 0.14, a semi-empirical factor (Refs. 3 and 4) is thus applied to the value of ν to give a more realistic result.

$$\nu_f = k.\nu \qquad ..(9)$$

where $k = 1.0$ for $\mu > 0.14$

and $k = 1.0 - 2.14\ (\mu - 0.14)$ for $\mu \leq 0.14$

2.3 Blade Flapping Motion

In steady flight, the blade-motion must be periodic and is therefore capable of being expressed in a Fourier series. Thus for rigid blades:

$$\beta = a_0 - a_1 \cos\psi - b_1 \sin\psi$$

ignoring terms of second order and above.

Thus a_0 represents the blade coning angle,

a_1 represents the first order longitudinal flapping angle,

b_1 represents the first order lateral flapping angle.

Reference 6 states that the higher harmonics of the Fourier series (a_2, b_2, a_3, etc.) represent blade displacements which are of the same order as elastic deflections of the rotor blades. Thus it is inconsistent to calculate the higher harmonics of rigid blade motion without including structural deflections of the blade.

2.4 Blade Element Angle of Attack

Figure 2 represents a typical blade element. Reference 7 shows that for small β, the velocity components at the rotor blade element may be expressed non-dimensionally as:

$$u_T = \frac{U_T}{\Omega R} = x + \mu \sin\psi$$

$$u_p = \frac{U_p}{\Omega R} = \lambda + \frac{1}{2} \mu a_1 + (-\mu a_0 + x b_1) \cos\psi$$

$$+ (-x a_1) \sin\psi + \frac{1}{2} \mu a_1 \cos 2\psi + \frac{1}{2} \mu b_1 \sin 2\psi$$

Also from figure 2,

$$\phi = \arctan \frac{U_p}{U_T}$$

Thus the local angle of attack of a blade element can be found from:

$$\alpha_r = \theta_0 + x\theta_1 + \phi \qquad ..(10)$$

2.5 Rotor Quantities Derived From Blade Element Theory

The extended analysis of Gessow and Crim (Ref. 5) has been used as the basis for all blade element equations presented here. This extended theory differs from standard rotor theory in that no limitation is placed on the inflow angle and the reversed velocity region is taken into account.* However, no allowances are made for blade stall outside of the reversed velocity region, and compressibility effects are ignored.

The equations presented here differ from those used in reference 5 in that:

(a) Second order flapping coefficients are neglected, for reasons stated earlier.

(b) Terms of high order in μ and θ_1 are neglected. High order terms in μ are ignored since above approximately $\mu = 0.3$, blade stall and compressibility effects become significant. As most blades in present day conventional helicopters have twist less than 12 deg, high order terms in θ_1 are ignored.

These modifications vastly simplify the equations and method of solution. By ignoring second order flapping, for example, the five simultaneous flapping equations of Ref. 5 reduce to three equations which can be solved directly.

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* When a helicopter is in forward flight, part of the inner section of the retreating blade is rotating in the aft direction at a speed which is lower than the forward speed. Thus air flow is in a reverse direction in this section, which is known as the reversed velocity region.

These are:

$$a_o = \frac{\gamma}{2} \left\{ \sin\theta_0 \left[\frac{B^3}{3} + \mu^2 \frac{B}{2} - \theta_1 \lambda \frac{B^3}{3} - \theta_1^2 \frac{B^5}{10} \right] \right.$$

$$+ \cos\theta_0 \left[\lambda \frac{B^2}{2} + \mu^2 \frac{\lambda}{8} + \theta_1 \left(\frac{B^4}{4} + \mu^2 \frac{B^2}{4} \right) - \theta_1^2 \lambda \frac{B^4}{8} \right]$$

$$\left. + \frac{\bar{c}_1}{a} \left[\frac{1}{8} \mu^2 |\lambda| + \frac{\mu^3}{9\pi} \right] + \frac{\bar{c}_{d_o}}{a} \left[\frac{\mu}{\pi} \lambda |\lambda| + \frac{\mu^2 \lambda}{16} \right] \right\} \quad ..(11)$$

$$a_1 = \frac{\cos\theta_0 \left(\mu \lambda \frac{B^2}{2} + \theta_1 \mu \frac{B^4}{2} \right) + \sin\theta_0 \left(\frac{2}{3} \mu B^3 - \theta_1 \mu \lambda \frac{B^3}{3} - \theta_1^2 \mu \frac{B^5}{5} \right)}{\cos\theta_0 \left(\frac{B^4}{4} - \frac{B^2}{8} \mu^2 \right) - \sin\theta_0 \left(\theta_1 \frac{B^5}{5} \right)} \quad ..(12)$$

$$b_1 = \frac{\cos\theta_0 \left(\mu \frac{B^3}{3} a_o \right) - \sin\theta_0 \left(\frac{B^4}{4} \mu \theta_1 a_o \right)}{\cos\theta_0 \left(\frac{B^4}{4} + \mu^2 \frac{B^2}{8} \right) - \sin\theta_0 \left(\theta_1 \frac{B^5}{5} \right)} \quad ..(13)$$

Reference 5 states that for most power-on flight conditions $\bar{c}_1 = 1.2$ and $\bar{c}_{d_o} = 1.1$ give satisfactory results.

The thrust equation of Ref. 5 is reduced to:

$$\frac{2C_T}{\sigma a} = \sin\theta_0 \left[\frac{B^3}{3} + \mu^2 \frac{B}{2} - \theta_1 \lambda \frac{B^3}{3} - \theta_1^2 \frac{B^5}{10} \right]$$

$$+ \cos\theta_0 \left[\lambda \frac{B^2}{2} + \mu^2 \frac{\lambda}{8} + \theta_1 \left(\frac{B^4}{4} + \mu^2 \frac{B^2}{4} \right) - \theta_1^2 \lambda \frac{B^4}{8} \right]$$

$$+ \frac{\bar{c}_1}{a} \left[\frac{1}{8} \mu^2 |\lambda| + \frac{\mu^3}{9\pi} \right] + \frac{\bar{c}_{d_o}}{a} \left[\frac{\mu}{\pi} \lambda |\lambda| + \frac{\mu^2 \lambda}{16} \right] \quad ..(14)$$

After likewise simplification, the profile power coefficient is given by:

$$\left(\frac{2}{\sigma} C_{P_0}\right) = K_1\left(\frac{1}{4} + \frac{3}{4}\mu^2\right) + K_2\left(\frac{1}{5} + \frac{1}{2}\mu^2\right) + K_3\left(\frac{1}{6} + \frac{3}{8}\mu^2\right)$$
$$+ K_4\left(\frac{1}{7} + \frac{3}{10}\mu^2\right) + \left(\lambda - \frac{1}{2}\mu a_1\right)\left(\frac{1}{3}K_7 + \frac{1}{4}K_8 + \frac{1}{5}K_9\right)$$
$$+ \frac{1}{2}\mu^2 \lambda K_7 + \lambda^2\left(\frac{1}{2}K_{19} + \frac{1}{3}K_{20}\right)$$
$$+ \left(\frac{1}{2}a_1^2 + \frac{1}{2}b_1^2\right)\left(\frac{1}{4}K_{19} + \frac{1}{5}K_{20}\right) \qquad ..(15)$$

where $K_1 = \delta_0 + \delta_1 \sin\theta_0 + \delta_2 \sin^2\theta_0$

$K_2 = K_7 \theta_1$

$K_3 = \frac{1}{2} K_8 \theta_1$

$K_4 = \frac{1}{3} K_9 \theta_1$

$K_5 = K_6 = 0$

$K_7 = \delta_1 \cos\theta_0 + \delta_2 \sin 2\theta_0$

$K_8 = (-\delta_1 \sin\theta_0 + 2\delta_2 \cos 2\theta_0)\theta_1$

$K_9 = (-\frac{1}{2}\delta_1 \cos\theta_0 - 2\delta_2 \sin 2\theta_0)\theta_1^2$

$K_{10} = K_{11} = K_{12} = 0$

$K_{13} = \delta_2 \cos^2\theta_0$

$K_{14} = -\delta_2 \sin(2\theta_0)\theta_1$

$K_{15} = K_{16} = K_{17} = K_{18} = 0$

$$K_{19} = \delta_0 + \delta_1 \sin\theta_0 + \delta_2 \cos^2\theta_0$$

$$K_{20} = (\delta_1 \cos\theta_0 - \delta_2 \sin 2\theta_0) \quad \theta_1$$

$$K_{21} = K_{22} = K_{23} = K_{24} = 0$$

Certain 'K' constants which are equated to zero result from neglect of high order terms in θ_1 and thus have been ignored. K_{13} and K_{14} are used in Ref. 5 to calculate the decelerating torque. Since accelerating and decelerating torque are not required for performance calculations of helicopters in power-on flight, they are not considered here.

2.6 Power Calculations

The power absorbed by a helicopter is made up of several components. The standard equations used to calculate these are shown below.

$$P_i = T \cdot \nu_f \qquad \text{(Induced)} \qquad ..(16)$$

$$P_p = \frac{1}{2} \rho V^3 f \qquad \text{(Parasite)} \qquad ..(17)$$

$$P_o = C_{P_o} [\rho \pi R^2 (\Omega R)^3] \qquad \text{(Profile)} \qquad ..(18)$$

where C_{P_o} is given by equation (15).

For convenience, parameter P_{sum} is defined as

$$P_{sum} = P_i + P_p + P_o \qquad ..(19)$$

Climb power P_c is calculated according to whether

(a) Climb angle γ_c is known, in which case Ref. 2 gives the following expression -

$$P_c = \sin\gamma_c \left[-\sin\gamma_c \frac{P_p}{T\Omega R} \frac{\cos\alpha}{\mu} + \left(1-\cos^2\gamma_c \frac{P_p{}^2}{T^2(\Omega R)^2} \frac{\cos^2\alpha}{\mu^2} \right)^{\frac{1}{2}} \right] \frac{\mu}{\cos\alpha} \cdot T\Omega R \qquad ..(20)$$

Conversely the climb angle may be determined if the available climb power is known.

(b) Available shaft power is known, in which case -

$$P_c = P_{avail} - P_{sum} \qquad ..(21)$$

The required power, P_{req}, is now defined as

$$P_{req} = P_{sum} + P_c \qquad ..(22)$$

2.7 Rate of Climb Calculation

Climb rate is given by the expression:

$$V_v = V \sin\gamma_c \qquad ..(23)$$

where V is flight path velocity.

In axial flight ($\mu = 0$) and, ignoring the parasite power component, we have

$$V_v = \frac{P_c}{W} \qquad ..(24)$$

2.8 Range and Endurance

These quantities are given by the Breguet Equations of Ref. 8 namely -

$$\text{Range} = \frac{T.V}{P_{sum}(SFC)} \left[- \text{Ln}\left(1 - \frac{W_f}{W} \right) \right] \qquad ..(25)$$

$$E = \frac{T}{P_{sum}(SFC)} \left[- Ln \left(1 - \frac{W_f}{W} \right) \right] \qquad ..(26)$$

Equations 25 and 26 assume that P_{sum}/T, V and SFC are constants independent of the helicopter weight.

3. PROGRAM STRUCTURE

This section gives a description of:

(1) Operating procedure for running 'POLAR'.

(2) Basic structure of calculations made for a particular airspeed.

(3) Estimating range/endurance.

(4) Creating speed-power polar.

(5) Estimating power for a given V and γ_c.

(6) Estimating γ_c for given V and shaft power.

3.1 Operating 'POLAR'

Program 'POLAR' provides four options:

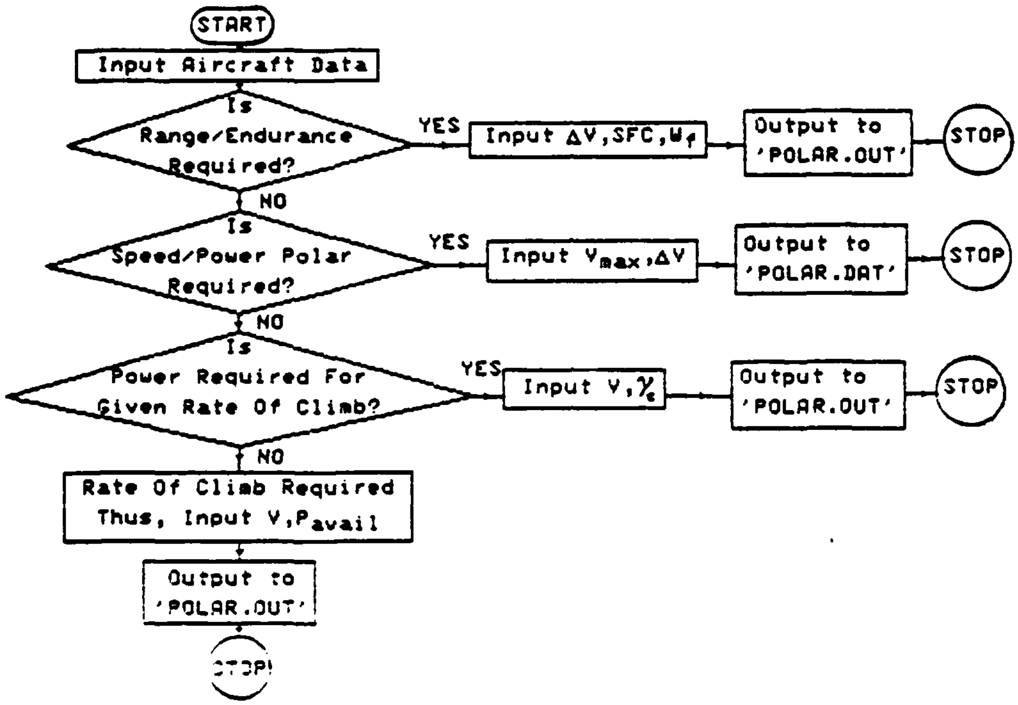
(i) Estimate range/endurance;

(ii) Provide a speed-power polar for level flight;

(iii) Estimate the power required for a given airspeed and rate of climb; and

(iv) Estimate the possible rate of climb for a given airspeed and shaft power.

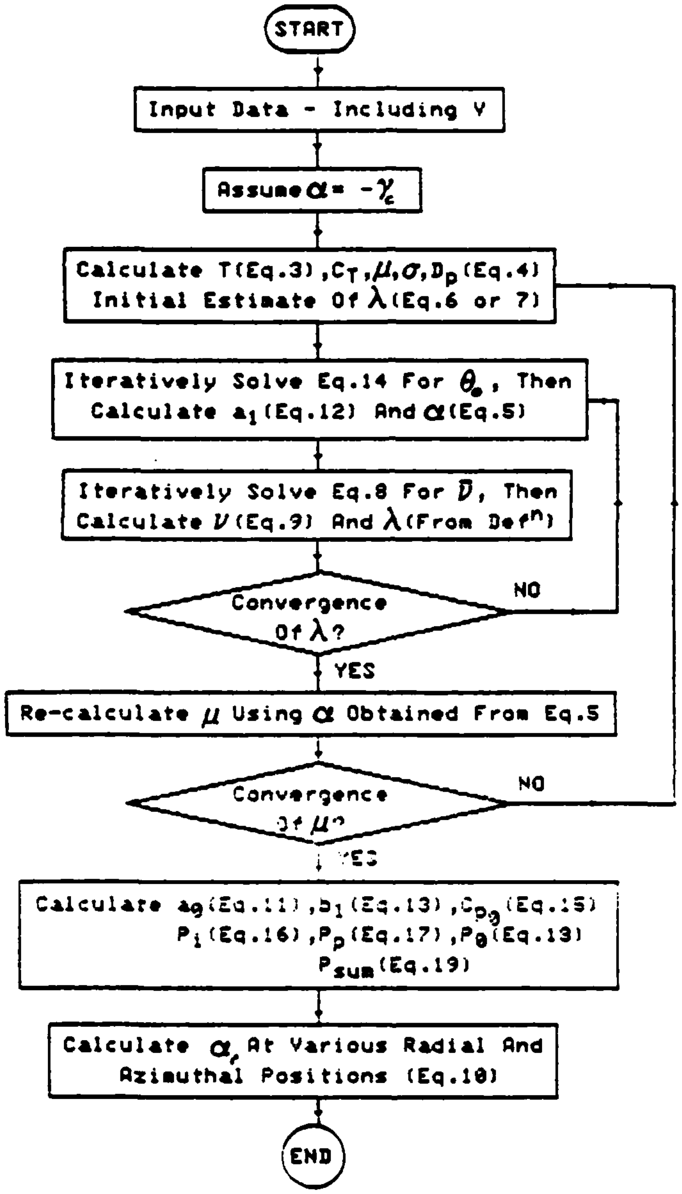
A flow chart showing the operating procedure by which 'POLAR' provides these options is shown below:



Detailed examples of running 'POLAR' are given in Section 4.

3.2 Basic Structure of Calculations

A flow chart describing the basic calculations which are made at a particular airspeed is shown below. This basic structure is used in each of the four program options.



Initially α is assumed to be equal to $-\gamma_c$. This results in quicker convergence of the iterative calculations and allows an initial estimate for μ. λ and θ_0 are also initially unknown and are both required in the blade-element relationships. Thus λ is obtained using simple momentum theory and θ_o is calculated. Using these values of μ, λ and θ_0, a_1 and then α are calculated. This value of α is then used to re-calculate λ and the process continues until λ converges.

Having determined a value for α, μ is recalculated and the process for determining λ, θ_o, a_1 and α is repeated. Again, this loop is repeated until convergence of μ.

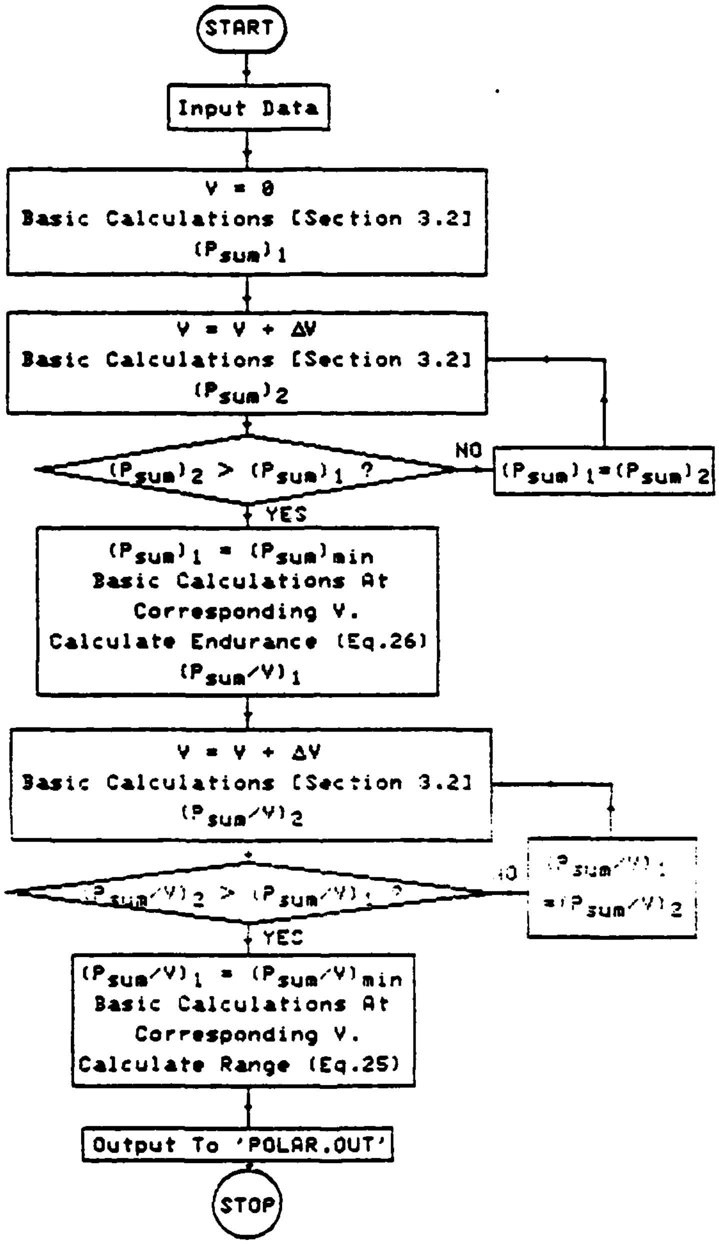
Since values of the required quantities have now been determined, the blade angles and power components are directly calculated.

3.3 Estimating Range and Endurance

In order to estimate the range and endurance, the Breguet equations (Eq. 25 and 26) are employed. Assuming that SFC is independent of velocity, the minimum fuel consumption per unit distance and hence the maximum range are achieved at the speed for minimum P_{sum}/V. Similarly, the maximum endurance is achieved at the speed for minimum P_{sum}.

'POLAR' determines range and endurance in a manner shown below.

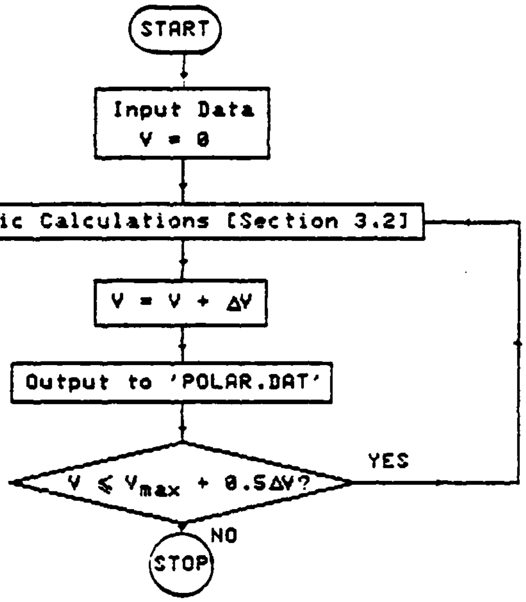
If range/endurance are required, it can be seen from section 3.1 that ΔV, SFC and W_f are required. Having typed in these parameters, 'POLAR' calculates P_{sum} at progressively higher velocities until a minimum is reached $(P_{sum})_{min}$, from whence the endurance is calculated. Similarly $(P_{sum}/V)_{min}$ is found and range calculated. Note that the velocity for maximum endurance is always lower than the velocity for maximum range.



3.4 Creating a Speed-Power Polar

Program 'POLAR' will create a binary data file which can be input to program 'TRANS' (App.2) in order to obtain graphical or tabular data over a range of airspeeds for a helicopter in level flight. This file is created when a speed-power polar is specified (Section 3.1).

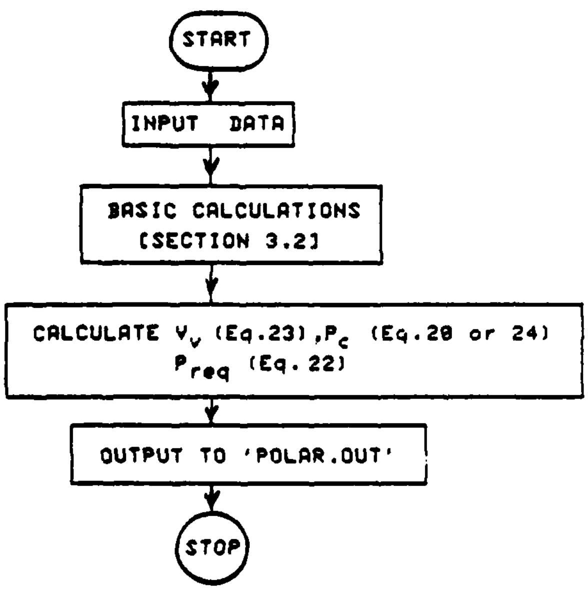
The calculations are accomplished as seen below.



The basic calculations are made, at each incremental velocity specified, starting at V=0 until the specified V_{max} is reached.

3.5 Estimating Power for a Given Velocity and Climb Angle

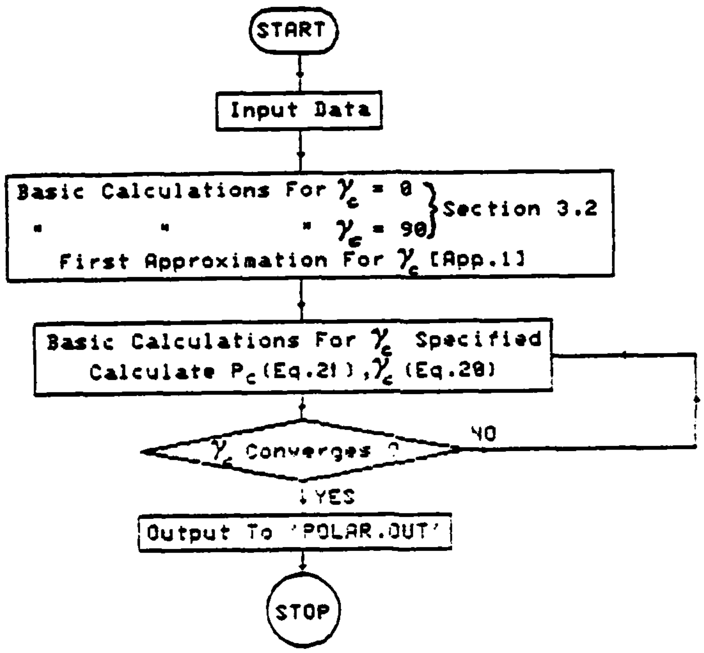
Power required for a given flight condition is estimated in the manner shown below.



Basic calculations are made at the velocity specified, then V_v, P_c and P_{req} are calculated directly.

3.6 Estimating Climb Angle for Given Velocity and Shaft Power

The climb angle, γ_c and thus rate of climb V_v are estimated as shown below.



A first approximation is made for γ_c, as seen in Appendix 1, and the basic calculations are made using this value for γ_c. P_c and then γ_c are calculated. If this value of γ_c does not agree with the first approximation, then this value is used in the basic calculations and the process repeated until γ_c converges.

4. EXAMPLE OF PERFORMANCE ESTIMATION

This section demonstrates the use of the blade-element analysis program 'POLAR' to estimate the following performance parameters for a hypothetical helicopter:

1. Speed-Power Polar in level flight.

2. Range and Endurance.

3. Climb Performance.

   3.1 Rate of climb possible for a given airspeed and shaft power.

   3.2 Shaft power required for a given airspeed and rate of climb.

   3.3 Maximum rate of climb.

   3.4 Power required for a given vertical climb.

4. Ceiling Estimates.

   4.1 Absolute ceiling.

   4.2 Service ceiling.

   4.3 Hover ceiling out of ground effect.

   4.4 Hover ceiling in ground effect.

The helicopter is assumed to have the following specifications:

$W = 9604$ lb (42718 N)

$W_f = 1439$ lb (6400 N)

$\Omega R = 717.2$ ft/s (218.6 m/s)

$R = 21$ ft (6.4 m)

$c = 1.3$ ft (0.395 m)

$b = 4$

$\theta_1 = -8°$

$\gamma = 15$

$B = 0.97$

$a = 5.73$/rad

$f = 17$ ft² (1.58 m²)

$P_{inst} = 1500$ Hp

SFC= 1.11 lb/Hp-Hr (0.504 kg/Hp-Hr)

The main rotor blades are assumed to be of NACA 23012 aerofoil section, with drag coefficient related to angle of attack by the following expression: $c_{d_o} = 0.0087 - 0.0216\ \alpha_r + 0.4\ \alpha_r{}^2$ therefore $\delta_0 = 0.0087$, $\delta_1 = -0.0216$, $\delta_2 = 0.4$.

It is also assumed that the helicopter is operating at ISA + 15°C conditions, unless otherwise stated. (Appendix 3)

Appendix 1 shows how program 'POLAR' is loaded onto a computer and explains how running 'POLAR' creates either a '.DAT' file or a '.OUT' file. A '.DAT' file requires further processing by program 'TRANS' (Appendix 2) to give column and graphical output.

4.1 Speed-Power Polar in Level Flight

For the example following, metric units are incorporated for the helicopter input data. When a Speed-Power polar is specified, the results are obtained over a range of speeds and output in binary form to file 'POLAR.DAT'. 'TRANS' can then be run to obtain a plot of power against speed (Fig. 3). Alternatively, 'TRANS' may be run to produce a column print of the same information, as shown in Appendix 2.

```
.RU POLAR

TITLE (TWO LINES OF UP TO 48 CHARACTERS)
:   BLADE ELEMENT METHOD
    POWER SPEED POLAR

ARE UNITS IN IMPERIAL OR METRIC (I OR M) ? M

ALL UP WEIGHT (N OR LB) = ? 42718

ROTOR TIP SPEED (M/S OR FT/S) = ? 218.6

ROTOR RADIUS (M OR FT) = ? 6.4

ROTOR BLADE CHORD (M OR FT) = ? 0.395

NUMBER OF ROTOR BLADES = ? 4

2D LIFT CURVE SLOPE (/RAD) = ? 5.73

TIP LOSS FACTOR = ? 0.97

BLADE TWIST (DEG) = ? -8

EQUIVALENT FLAT PLATE AREA (M**2 OR FT**2) = ? 1.58

LOCKS NUMBER = ? 15

DRAG POLAR CO-EFFICIENT (DEL0) = ? 0.0087

DELTA1 (/RAD) = ? -0.0216

DELTA2 (/RAD**2) = ? 0.4

AIR DENSITY (KG/M**3 OR SLUG/FT**3) = ? 1.1644

IS RANGE/ENDURANCE REQUIRED ? (Y OR N) : N

IS A SPEED-POWER POLAR REQUIRED ? (Y OR N) : Y

MAXIMUM VELOCITY (KNOTS) = ? 150

VELOCITY INTERVAL (KNOTS) = ? 2


END OF EXECUTION
CPU TIME: 6.10   ELAPSED TIME: 53.40
EXIT

.
```

4.2 Range and Endurance

The example below shows how 'POLAR' is run to obtain the range of the hypothetical helicopter. The data is input in metric units and the results are output to file 'POLAR.OUT'. 'POLAR' is run as in the previous example, with the following changes.

```
IS RANGE/ENDURANCE REQUIRED ? (Y OR N) : Y

VELOCITY INTERVAL (KNOTS) = ? 2.0

SPECIFIC FUEL CONSUMPTION ? (KG/HP-HR OR LB/HP-HR) : 0.504

WEIGHT OF FUEL CARRIED ? (N OR LB) : 6400

STOP

END OF EXECUTION
CPU TIME: 3.60  ELAPSED TIME: 55.02
EXIT



 .TY POLAR.OUT
 BLADE ELEMENT METHOD
 RANGE/ENDURANCE ESTIMATE
  ALL UP WEIGHT =      42718.0 N
  AIR DENSITY =  1.1644 KG/M**3
  TIP SPEED =  218.6 M/S
  ROTOR RADIUS =  6.4 M
  ROTOR CHORD =  0.395 M
  NUMBER OF ROTOR BLADES =  4.0
  LIFT CURVE SLOPE =  5.73 /RAD
  TIP LOSS FACTOR =  0.97
  BLADE TWIST =    -8.0 DEG
  EQUIVALENT FLAT PLATE AREA =  1.58 M**2
  LOCKS NUMBER =  15.0
  DRAG POLAR CO-EFFICIENT(DELTA 0)        =  0.0087
  DELTA 1 = -0.0216 /RAD
  DELTA 2 =  0.4000 /RAD**2
  S.F.C. =  0.504 KG/HP-HR
  FUEL WEIGHT =  6400.0 N

  RESULTS FOR ENDURANCE OF  2.81 HOURS, ARE AS FOLLOWS :

  AIRSPEED =   72.00 KNOTS
  ADVANCE RATIO(MU) =  0.169
  INDUCED VELOCITY(NU) =   3.30 M/S
  INFLOW RATIO(LAMBDA) =  -0.0306
  FLAT PLATE DRAG =    1262.0 N
  THRUST =  42736.6 N
  THRUST CO-EFFICIENT (CT) =  0.00597
  BLADE SOLIDITY =  0.0786
  COLLECTIVE (THETA 0) =  13.1 DEG
  CONING ANGLE (a0) =   7.8 DEG
  LONGITUDINAL FLAPPING ANGLE (a1) =   2.8 DEG
  LATERAL FLAPPING ANGLE (b1) =   1.8 DEG
  DISC ANGLE OF ATTACK =  -4.5 DEG
  INDUCED POWER =   217.8 HP
  PARASITE POWER =    62.7 HP
  PROFILE POWER =   220.1 HP
  TOTAL POWER =   500.6 HP
  LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =   7.2 DEG
  LOCAL ANGLE OF ATTACK(@270DEG&TIP) =   6.4 DEG
  LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =   8.2 DEG
  RADIAL POINT OF UT=0.4 (@270DEG) =  0.57

 1 RESULTS FOR RANGE OF    471.5 KM, ARE AS FOLLOWS :

  AIRSPEED =  112.00 KNOTS
  ADVANCE RATIO(MU) =  0.261
  INDUCED VELOCITY(NU) =   2.46 M/S
  INFLOW RATIO(LAMBDA) =  -0.0510
  FLAT PLATE DRAG =   3053.8 N
  THRUST =  42827.0 N
  THRUST CO-EFFICIENT (CT) =  0.00598
  BLADE SOLIDITY =  0.0786
  COLLECTIVE (THETA 0) =  14.3 DEG
  CONING ANGLE (a0) =   7.8 DEG
  LONGITUDINAL FLAPPING ANGLE (a1) =   4.6 DEG
  LATERAL FLAPPING ANGLE (b1) =   2.7 DEG
  DISC ANGLE OF ATTACK =  -8.7 DEG
  INDUCED POWER =   141.4 HP
  PARASITE POWER =   236.0 HP
  PROFILE POWER =   241.7 HP
  TOTAL POWER =   619.1 HP
  LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =   3.3 DEG
  LOCAL ANGLE OF ATTACK(@270DEG&TIP) =   8.5 DEG
  LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =   9.2 DEG
```

4.3 Climb Performance

In order to estimate climb performance using 'POLAR', the power available at the main-rotor shaft must be known. The installed engine power of the hypothetical helicopter is given as 1500 Hp. However, not all of this power is available at the main rotor shaft. Some power will be absorbed by the main-rotor transmission, accessories (hydraulics, pneumatics, electronics, heating and cooling systems etc.) and the tail-rotor. We will assume that the sum of these power losses is 15% of the installed engine power.

It should be noted that in low speed forward flight, the power absorbed by the tail-rotor is proportional to the power required by the main-rotor which in turn varies with forward speed (Fig. 3). Thus in reality the power losses will not be constant at various velocities. (Ref. 6 states that tail-rotor losses vary from 6% of total power in hover, to 3% in forward flight). For simplicity we will assume 15% losses throughout the speed range. Therefore, the available power at the main rotor shaft is 1275 Hp.

It should also be noted that in fast descents (negative climbs). where $|V_v| \geq \nu$, helicopters may enter the 'Vortex Ring State'. When this phenomenon occurs, momentum theory breaks down and 'POLAR' will not give meaningful results.

4.3.1 Rate of climb for a given airspeed and shaft power

In the example below, 'POLAR' is used to estimate the rate of climb at a flight path airspeed of 50 kt. The input data is in Imperial units.

```
.RU POLAR

TITLE (TWO LINES OF UP TO 60 CHARACTERS)
: BLADE ELEMENT METHOD - IMPERIAL UNITS
RATE OF CLIMB REQUIRED FOR GIVEN AIRSPEED & SHAFT POWER

ARE UNITS IN IMPERIAL OR METRIC (I OR M) ? I

ALL UP WEIGHT (N OR LB) = ? 9604

ROTOR TIP SPEED (M/S OR FT/S) = ? 717.2

ROTOR RADIUS (M OR FT) = ? 21

ROTOR BLADE CHORD (M OR FT) = ? 1.3

NUMBER OF ROTOR BLADES = ? 4

2D LIFT CURVE SLOPE (/RAD) = ? 5.73

TIP LOSS FACTOR = ? 0.97

BLADE TWIST (DEG) = ? -8

EQUIVALENT FLAT PLATE AREA (M**2 OR FT**2) = ? 17

LOCKS NUMBER = ? 15

DRAG POLAR CO-EFFICIENT (DEL0) = ? 0.0087

DELTA1 (/RAD) = ? -0.0216

DELTA2 (/RAD**2) = ? 0.4

AIR DENSITY (KG/M**3 OR SLUG/FT**3) = ? 0.002259

IS RANGE/ENDURANCE REQUIRED ? (Y OR N) : N

IS A SPEED-POWER POLAR REQUIRED ? (Y OR N) : N

IS POWER REQUIRED FOR A GIVEN CLIMB ANGLE
i.e. RATE OF CLIMB (Y OR N) ? N

VELOCITY IN KNOTS ? : 50.

WHAT IS POWER AVAILABLE AT MAIN ROTOR SHAFT (HP) ? : 1275

STOP

END OF EXECUTION
CPU TIME: 1.65  ELAPSED TIME: 1:6.02
EXIT
```

The results of this run, are output onto File 'POLAR.OUT'.

```
.TY POLAR.OUT
BLADE ELEMENT METHOD - IMPERIAL UNITS
RATE OF CLIMB REQUIRED FOR GIVEN AIRSPEED & SHAFT POWER
 ALL UP WEIGHT =       9604.0 LB
 AIR DENSITY =   0.002259 SLUG/FT**3
 TIP SPEED =   717.2 FT/S
 ROTOR RADIUS =   21.0 FT
 ROTOR CHORD =   1.3 FT
 NUMBER OF ROTOR BLADES =   4.0
 LIFT CURVE SLOPE =   5.73 /RAD
 TIP LOSS FACTOR =   0.97
 BLADE TWIST =   -8.0 DEG
 EQUIVALENT FLAT PLATE AREA =   17.0 FT**2
 LOCKS NUMBER =   15.0
 DRAG POLAR CO-EFFICIENT (DELTA 0) =   0.0087
 DELTA 1 = -0.0216 /RAD
 DELTA 2 =   0.4000 /RAD**2
 AIRSPEED =   50.00 KNOTS

 ADVANCE RATIO(MU) =   0.100
 INDUCED VELOCITY(NU) =   17.83 FT/S
 INFLOW RATIO(LAMBDA) =   -0.0875
 FLAT PLATE DRAG =     136.7 LB
 THRUST =    [illegible] LB
 THRUST CO-EFFICIENT (CT) =   0.00631
 BLADE SOLIDITY =   0.0788
 COLLECTIVE (THETA 0) =   18.3 DEG
 CONING ANGLE (a0) =    8.9 DEG
 LONGITUDINAL FLAPPING ANGLE (a1) =    2.4 DEG
 LATERAL FLAPPING ANGLE (b1) =    1.2 DEG
 DISC ANGLE OF ATTACK = -32.1 DEG
 INDUCED POWER =    313.5 HP
 PARASITE POWER =     21.0 HP
 PROFILE POWER =    225.1 HP
 TOTAL POWER =    559.6 HP
 REQUIRED/AVAILABLE SHAFT POWER =   1275.0 HP
 CLIMB POWER =    715.4 HP
 RATE OF CLIMB =   2458.1 FT/MIN
 CLIMB ANGLE  =   29.0 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =    1.8 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&TIP) =    7.4 DEG
 LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =    4.8 DEG
 RADIAL POINT OF UT=0.4 (@270DEG) =   0.500
```

4.3.2 Shaft power required for a given airspeed and climb rate

From the previous example we can see that at an airspeed of 50 kts, power of 1275 Hp, the rate of climb is 2458.1 ft/min and climb angle is 29°.

The inverse case is shown in the example below, where 'POLAR' is used to estimate the power required at 50 kt and climb angle of 29°. 'POLAR' is run as in the previous example, with the following changes.

```
IS POWER REQUIRED FOR A GIVEN CLIMB ANGLE
i.e. RATE OF CLIMB (Y OR N) ? Y

VELOCITY IN KNOTS ? : 50.

WHAT IS CLIMB ANGLE - GAMMA = ASIN(VV/V) - (DEG) ? : 29

STOP

END OF EXECUTION
CPU TIME: 1.44  ELAPSED TIME: 50.60
EXIT
```

Results typed below agree with those of the previous example. Any small differences result from rounding errors.

```
.TY POLAR.OUT
BLADE ELEMENT METHOD - IMPERIAL UNITS
POWER REQUIRED FOR A GIVEN AIRSPEED & CLIMB ANGLE
 ALL UP WEIGHT =      9604.0 LB
 AIR DENSITY =  0.002259 SLUG/FT**3
 TIP SPEED =  717.2 FT/S
 ROTOR RADIUS =  21.0 FT
 ROTOR CHORD =  1.3 FT
 NUMBER OF ROTOR BLADES =  4.0
 LIFT CURVE SLOPE =  5.73 /RAD
 TIP LOSS FACTOR =  0.97
 BLADE TWIST =  -8.0 DEG
 EQUIVALENT FLAT PLATE AREA =  17.0 FT**2
 LOCKS NUMBER =  15.0
 DRAG POLAR CO-EFFICIENT (DELTA 0) =  0.0087
 DELTA 1 = -0.0216 /RAD
 DELTA 2 =  0.4000 /RAD**2
 AIRSPEED =  50.00 KNOTS

 ADVANCE RATIO(MU) =  0.100
 INDUCED VELOCITY(NU) =  17.83 FT/S
 INFLOW RATIO(LAMBDA) =  -0.0974
 FLAT PLATE DRAG =     136.7 LB
 THRUST =    [illegible]71.0 LB
 THRUST CO-EFFICIENT (CT) =  0.00601
 BLADE SOLIDITY =  0.0788
 COLLECTIVE (THETA 0) =  18.3 DEG
 CONING ANGLE (a0) =   8.9 DEG
 LONGITUDINAL FLAPPING ANGLE (a1) =   2.4 DEG
 LATERAL FLAPPING ANGLE (b1) =   1.2 DEG
 DISC ANGLE OF ATTACK = -32.1 DEG
 INDUCED POWER =   313.5 HP
 PARASITE POWER =     21.0 HP
 PROFILE POWER =   225.1 HP
 TOTAL POWER =   559.6 HP
 REQUIRED/AVAILABLE SHAFT POWER =  1274.0 HP
 CLIMB POWER =   714.4 HP
 RATE OF CLIMB =  2454.8 FT/MIN
 CLIMB ANGLE  =  29.0 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =    1.8 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&TIP) =    7.4 DEG
 LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =    4.8 DEG
 RADIAL POINT OF UT=0.4 (@270DEG) =  0.500
```

4.3.3 Maximum rate of climb

The maximum rate of climb occurs when the power required for level flight is a minimum. Using results of §4.2 or figure 3, 72 kt is the velocity at which maximum climb rate will be achieved. This is demonstrated in the example below. 'POLAR' is run as in §4.3.1 but with a velocity of 72 knots. Only the output file is shown.

```
.TY POLAR.OUT
BLADE ELEMENT METHOD - IMPERIAL UNITS
MAXIMUM RATE OF CLIMB
 ALL UP WEIGHT =      9604.0 LB
 AIR DENSITY =  0.002259 SLUG/FT**3
 TIP SPEED =  717.2 FT/S
 ROTOR RADIUS =  21.0 FT
 ROTOR CHORD =  1.3 FT
 NUMBER OF ROTOR BLADES =  4.0
 LIFT CURVE SLOPE =  5.73 /RAD
 TIP LOSS FACTOR =  0.97
 BLADE TWIST =  -8.0 DEG
 EQUIVALENT FLAT PLATE AREA =  17.0 FT**2
 LOCKS NUMBER =  15.0
 DRAG POLAR CO-EFFICIENT (DELTA 0) =  0.0087
 DELTA 1 = -0.0216 /RAD
 DELTA 2 =  0.4000 /RAD**2
 AIRSPEED =  72.00 KNOTS

 ADVANCE RATIO(MU) =  0.152
 INDUCED VELOCITY(NU) =  12.18 FT/S
 INFLOW RATIO(LAMBDA) =  -0.0923
 FLAT PLATE DRAG =    283.6 LB
 THRUST =   9710.1 LB
 THRUST CO-EFFICIENT (CT) =  0.00603
 BLADE SOLIDITY =  0.0788
 COLLECTIVE (THETA 0) =  18.4 DEG
 CONING ANGLE (a0) =   8.8 DEG
 LONGITUDINAL FLAPPING ANGLE (a1) =   3.7 DEG
 LATERAL FLAPPING ANGLE (b1) =   1.8 DEG
 DISC ANGLE OF ATTACK = -26.4 DEG
 INDUCED POWER =   215.0 HP
 PARASITE POWER =    62.7 HP
 PROFILE POWER =   231.9 HP
 TOTAL POWER =   509.5 HP
 REQUIRED/AVAILABLE SHAFT POWER =  1275.0 HP
 CLIMB POWER =   765.5 HP
 RATE OF CLIMB =   2630.2 FT/MIN
 CLIMB ANGLE  =  21.1 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =   0.2 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&TIP) =   8.5 DEG
 LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =   5.9 DEG
 RADIAL POINT OF UT=0.4 (@270DEG) =  0.552
```

4.3.4 Power required for a vertical climb

Vertical climb performance can be estimated by setting the climb angle to 90° and specifying the rate of climb. If the vertical rate of climb is required for a given power, the program must be run several times. Each time the flight path velocity must be varied until the power is equal to that available in the helicopter.

The example below shows the final result for the maximum vertical rate of climb.

```
.TY POLAR.OUT
BLADE ELEMENT METHOD - IMPERIAL UNITS
POWER REQUIRED FOR A VERTICAL CLIMB
 ALL UP WEIGHT =      9604.0 LB
 AIR DENSITY =  0.002259 SLUG/FT**3
 TIP SPEED =  717.2 FT/S
 ROTOR RADIUS =  21.0 FT
 ROTOR CHORD =  1.3 FT
 NUMBER OF ROTOR BLADES =  4.0
 LIFT CURVE SLOPE =  5.73 /RAD
 TIP LOSS FACTOR =  0.97
 BLADE TWIST =  -8.0 DEG
 EQUIVALENT FLAT PLATE AREA =  17.0 FT**2
 LOCKS NUMBER =  15.0
 DRAG POLAR CO-EFFICIENT (DELTA 0) =  0.0087
 DELTA 1 = -0.0216 /RAD
 DELTA 2 =  0.4000 /RAD**2
 AIRSPEED =  12.30 KNOTS

 ADVANCE RATIO(MU) =  0.000
 INDUCED VELOCITY(NU) =  38.79 FT/S
 INFLOW RATIO(LAMBDA) =  -0.0842
 FLAT PLATE DRAG =       9.0 LB
 THRUST =   9613.0 LB
 THRUST CO-EFFICIENT (CT) =  0.00597
 BLADE SOLIDITY =  0.0788
 COLLECTIVE (THETA 0) =  18.2 DEG
 CONING ANGLE (a0) =   8.9 DEG
 LONGITUDINAL FLAPPING ANGLE (a1) =   0.0 DEG
 LATERAL FLAPPING ANGLE (b1) =   0.0 DEG
 DISC ANGLE OF ATTACK = -90.0 DEG
 INDUCED POWER =   678.0 HP
 PARASITE POWER =      0.4 HP
 PROFILE POWER =   219.3 HP
 TOTAL POWER =   897.6 HP
 REQUIRED/AVAILABLE SHAFT POWER =  1274.8 HP
 CLIMB POWER =   377.2 HP
 RATE OF CLIMB =  1296.2 FT/MIN
 CLIMB ANGLE  =  90.0 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =   3.1 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&TIP) =   5.4 DEG
 LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =   3.1 DEG
 RADIAL POINT OF UT=0.4 (@270DEG) =  0.400
```

As can be seen above, the power required for a vertical climb rate of 1296.2 ft/min (1274.8 Hp) is almost equal to the power available (1275 Hp).

4.4 Ceiling Estimates

The helicopter performance analyst is interested in four ceilings, namely:

(1) Absolute Ceiling - defined as the maximum altitude at which the climb rate is zero.

(2) Service Ceiling - defined as the maximum altitude at which the climb rate is 100 ft/min (0.5 m/s).

(3) Hover Ceiling O.G.E. - defined as the maximum altitude at which the power available equals the power required to hover out of ground effect.

(4) Hover Ceiling I.G.E. - defined as the maximum altitude at which the power available equals the power required to hover in ground effect.

Reference 9 presents curves of engine power variation with altitude, which can be approximated by the following relationship:

$$P_{avail} = P_{SL} \; (1.13 \; \Delta - 0.13)$$

where P_{SL} is the shaft power available at sea-level (1275 Hp)

and

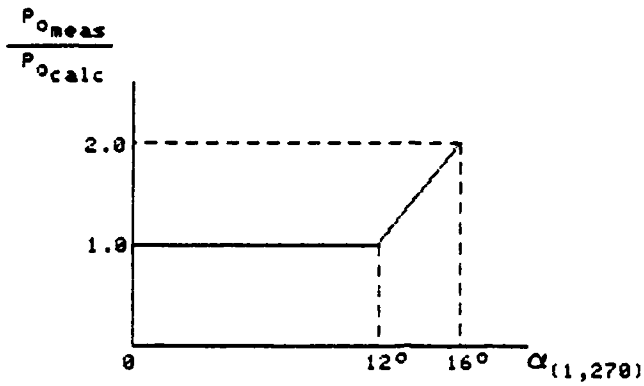
$$\Delta = \rho/\rho_{SL_{ISA+15}}$$

[It should be noted that actual engine power variation with altitude should be used, if known].

Variations of density with atmospheric conditions may be found using the locally available program 'ATMOS' (see Appendix 3).

At high altitudes, conditions are such that the main rotor may operate at blade angles of attack where partial stall may occur. Gustafson and Gessow (Ref. 10) found by flight tests that the ratio of measured profile power ($P_{o_{meas}}$) to calculated profile power ($P_{o_{calc}}$) was unity until $\alpha_{(1,270)}$ reached a value of 12°. This ratio rose linearly

with $\alpha_{(1,270)}$ to a value of approximately two when $\alpha_{(1,270)}$ reached 16°. (see diagram below). Above this value the effects of stall were such that the helicopter became too difficult to control.



4.4.1 Absolute Ceiling

Absolute ceiling will be attained when the minimum power in level flight is equal to the power available. 'POLAR' is run at a series of altitudes (densities) using the RANGE/ENDURANCE option, since minimum power is calculated in order to estimate endurance. [Note: If W_f and S.F.C. are unknown, arbitrary values may be assigned to these parameters, since they are only used to calculate range/ endurance and have no bearing on the power calculations]. At each altitude the minimum power is obtained. The value of $\alpha_{(1,270)}$ is then examined and, if necessary, P_o is adjusted to allow for stall as stated previously. Thus the true total power is obtained and then compared with the power available at that altitude. The process described above is repeated until convergence is achieved.

The example below is the final result obtained after several iterations and corresponds to an altitude of 17300 ft at ISA + 15°C conditions.

```
.RU POLAR

TITLE (TWO LINES OF UP TO 60 CHARACTERS)
:            BLADE ELEMENT METHOD
            ABSOLUTE CEILING ESTIMATION

ARE UNITS IN IMPERIAL OR METRIC (I OR M) ? I

ALL UP WEIGHT (N OR LB) = ? 9604

ROTOR TIP SPEED (M/S OR FT/S) = ? 717.2

ROTOR RADIUS (M OR FT) = ? 21

ROTOR BLADE CHORD (M OR FT) = ? 1.3

NUMBER OF ROTOR BLADES = ? 4

2D LIFT CURVE SLOPE (/RAD) = ? 5.73

TIP LOSS FACTOR = ? 0.97

BLADE TWIST (DEG) = ? -8

EQUIVALENT FLAT PLATE AREA (M**2 OR FT**2) = ? 17

LOCKS NUMBER = ? 15

DRAG POLAR CO-EFFICIENT (DEL0) = ? 0.0087

DELTA1 (/RAD) = ? -0.0216

DELTA2 (/RAD**2) = ? 0.4

AIR DENSITY (KG/M**3 OR SLUG/FT**3) = ? 0.001309

IS RANGE/ENDURANCE REQUIRED ? (Y OR N) : Y

VELOCITY INTERVAL (KNOTS) = ? 2

SPECIFIC FUEL CONSUMPTION ? (KG/HP-HR OR LB/HP-HR) : 1.11

WEIGHT OF FUEL CARRIED ? (N OR LB) : 1439

STOP

END OF EXECUTION
CPU TIME: 4.99  ELAPSED TIME: 1:56.78
EXIT

.
```

```
.TY POLAR.OUT
          BLADE ELEMENT METHOD
           ABSOLUTE CEILING ESTIMATION
 ALL UP WEIGHT =        9604.0 LB
 AIR DENSITY =   0.001309 SLUG/FT**3
 TIP SPEED =   717.2 FT/S
 ROTOR RADIUS =   21.0 FT
 ROTOR CHORD =   1.3 FT
 NUMBER OF ROTOR BLADES =  4.0
 LIFT CURVE SLOPE =   5.73 /RAD
 TIP LOSS FACTOR =   0.97
 BLADE TWIST =   -8.0 DEG
 EQUIVALENT FLAT PLATE AREA =   17.0 FT**2
 LOCKS NUMBER =   15.0
 DRAG POLAR CO-EFFICIENT (DELTA 0) =   0.0087
 DELTA 1 = -0.0216 /RAD
 DELTA 2 =   0.4000 /RAD**2
 S.F.C. =    1.11 LB/HP-HR
 FUEL WEIGHT =   1439.0 LB

 RESULTS FOR ENDURANCE OF  2.49 HOURS, ARE AS FOLLOWS :

 AIRSPEED =    94.00 KNOTS
 ADVANCE RATIO(MU) =   0.219
 INDUCED VELOCITY(NU) =   16.38 FT/S
 INFLOW RATIO(LAMBDA) =   -0.0530
 FLAT PLATE DRAG =      280.1 LB
 THRUST =    9608.1 LB
 THRUST CO-EFFICIENT (CT) =   0.01030
 BLADE SOLIDITY =   0.0788
 COLLECTIVE (THETA 0) =   18.1 DEG
 CONING ANGLE (a0) =   13.8 DEG
 LONGITUDINAL FLAPPING ANGLE (a1) =   [illegible] DEG
 LATERAL FLAPPING ANGLE (b1) =    4.0 DEG
 DISC ANGLE OF ATTACK =  -7.3 DEG
 INDUCED POWER =    286.2 HP
 PARASITE POWER =      80.8 HP
 PROFILE POWER =    196.7 HP
 TOTAL POWER =    563.7 HP
 LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =   11.7 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&TIP) =  14.1 DEG
 LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =   15.1 DEG
 RADIAL POINT OF UT=0.4 (@270DEG) =   0.619

1 RESULTS FOR RANGE OF    335.4 MILES, ARE  AS FOLLOWS :

 AIRSPEED =   142.00 KNOTS
 ADVANCE RATIO(MU) =   0.325
 INDUCED VELOCITY(NU) =   10.94 FT/S
 INFLOW RATIO(LAMBDA) =   -0.0935
 FLAT PLATE DRAG =      639.1 LB
 THRUST =    9625.2 LB
 THRUST CO-EFFICIENT (CT) =   0.01032
 BLADE SOLIDITY =   0.0788
 COLLECTIVE (THETA 0) =   20.1 DEG
 CONING ANGLE (a0) =   13.6 DEG
 LONGITUDINAL FLAPPING ANGLE (a1) =   9.7 DEG
 LATERAL FLAPPING ANGLE (b1) =    5.8 DEG
 DISC ANGLE OF ATTACK = -13.5 DEG
 INDUCED POWER =    191.5 HP
 PARASITE POWER =    278.5 HP
 PROFILE POWER =    215.7 HP
 TOTAL POWER =    685.7 HP
 LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =   -1.9 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&TIP) =   18.5 DEG
 LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =   18.5 DEG
 RADIAL POINT OF UT=0.4 (@270DEG) =   0.725
```

From these results, the minimum power occurs at 94 knots:

$$P_{total} = 563.7 \text{ Hp}, \; P_{o_{calc}} = 196.7 \text{ Hp}, \; \alpha_{(1,270)} = 14.1°$$

Now,

$$P_{req} = P_{total} + P_{o_{meas}} - P_{o_{calc}}$$

and

$$P_{o_{meas}} = P_{o_{calc}} + \left(\frac{\alpha_{(1,270)} - 12°}{4°} \right) P_{o_{calc}}$$

thus

$$P_{req} = P_{total} + \left(\frac{\alpha_{(1,270)} - 12°}{4°} \right) P_{o_{calc}}$$

$$= 563.7 + \left(\frac{14.1 - 12°}{4°} \right) 196.7$$

$$= 667 \text{ Hp}$$

Power available at the shaft is given by

$$P_{avail} = 1275 \; (1.13\text{x}0.001309/0.002259-0.13)$$

$$= 669 \text{ Hp}$$

Since the power available is approximately equal to the power required, the absolute ceiling is approximately 17,300 ft.

4.4.2 Service ceiling

The climb rate for this case is defined to be 100 ft/min (1.67 ft/s). As for 4.4.1, 'POLAR' must be run for a series of altitudes (densities) for level flight conditions. Having determined the velocity for minimum total power, γ_c can be calculated from equation (23). 'POLAR' can then be run using the specified γ_c and V. This process is continued until convergence of total power required and power available.

The example below shows how to estimate service ceiling and is the final result after several iterations.

'POLAR' is run as for the previous example, to find the velocity for minimum power.

The output file shown below corresponds to an altitude of 16900 ft.

```
.TY POLAR.OUT
                    SERVICE CEILING
                VELOCITY @ MINIMUM POWER
 ALL UP WEIGHT =      9604.0 LB
 AIR DENSITY =   0.001327 SLUG/FT**3
 TIP SPEED =  717.2 FT/S
 ROTOR RADIUS =   21.0 FT
 ROTOR CHORD =   1.3 FT
 NUMBER OF ROTOR BLADES =   4.0
 LIFT CURVE SLOPE =   5.73 /RAD
 TIP LOSS FACTOR =  0.97
 BLADE TWIST =   -8.0 DEG
 EQUIVALENT FLAT PLATE AREA =   17.0 FT**2
 LOCKS NUMBER =  15.0
 DRAG POLAR CO-EFFICIENT (DELTA 0) =   0.0087
 DELTA 1 = -0.0216 /RAD
 DELTA 2 =   0.4000 /RAD**2
 S.F.C. =    1.11 LB/HP-HR
 FUEL WEIGHT =   1439.0 LB

 RESULTS FOR ENDURANCE OF  2.50 HOURS, ARE AS FOLLOWS :

 AIRSPEED =    92.00 KNOTS
 ADVANCE RATIO(MU) =   0.215
 INDUCED VELOCITY(NU) =   16.51 FT/S
 INFLOW RATIO(LAMBDA) =   -0.0516
 FLAT PLATE DRAG =      272.0 LB
 THRUST =     9607.8 LB
 THRUST CO-EFFICIENT (CT) =   0.01016
 BLADE SOLIDITY =   0.0788
 COLLECTIVE (THETA 0) =   17.9 DEG
 CONING ANGLE (a0) =   13.6 DEG
 LONGITUDINAL FLAPPING ANGLE (a1) =    6.0 DEG
 LATERAL FLAPPING ANGLE (b1) =     3.9 DEG
 DISC ANGLE OF ATTACK =   -7.6 DEG
 INDUCED POWER =     288.3 HP
 PARASITE POWER =      76.8 HP
 PROFILE POWER =     195.8 HP
 TOTAL POWER =     560.9 HP
 LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =   11.6 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&TIP) =   13.7 DEG
 LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =   14.7 DEG
 RADIAL POINT OF UT=0.4 (@270DEG) =   0.615

1 RESULTS FOR RANGE OF    334.8 MILES, ARE  AS FOLLOWS :

 AIRSPEED =   142.00 KNOTS
 ADVANCE RATIO(MU) =   0.325
 INDUCED VELOCITY(NU) =   10.80 FT/S
 INFLOW RATIO(LAMBDA) =   -0.0930
 FLAT PLATE DRAG =      647.9 LB
 THRUST =     9625.8 LB
 THRUST CO-EFFICIENT (CT) =   0.01018
 BLADE SOLIDITY =   0.0788
 COLLECTIVE (THETA 0) =   19.9 DEG
 CONING ANGLE (a0) =   13.4 DEG
 LONGITUDINAL FLAPPING ANGLE (a1) =    9.6 DEG
 LATERAL FLAPPING ANGLE (b1) =     5.7 DEG
 DISC ANGLE OF ATTACK = -13.5 DEG
 INDUCED POWER =     189.0 HP
 PARASITE POWER =     282.3 HP
 PROFILE POWER =     215.7 HP
 TOTAL POWER =     687.0 HP
 LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =   -2.2 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&TIP) =   18.3 DEG
 LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =   18.2 DEG
 RADIAL POINT OF UT=0.4 (@270DEG) =   0.725
```

From these results, it can be seen that the velocity for minimum power is 92 knots (155.3 ft/s).

From equation (23),

$$\gamma_c = \arcsin (V_v/V) = \arcsin (1.67/155.3) = 0.615°$$

'POLAR' is then run as shown below:

```
.RU POLAR

TITLE (TWO LINES OF UP TO 60 CHARACTERS)
: SERVIVE CEILING ESTIMATE

ARE UNITS IN IMPERIAL OR METRIC (I OR M) ? I
ALL UP WEIGHT (N OR LB) = ? 9604
ROTOR TIP SPEED (M/S OR FT/S) = ? 717.2
ROTOR RADIUS (M OR FT) = ? 21
ROTOR BLADE CHORD (M OR FT) = ? 1.3
NUMBER OF ROTOR BLADES = ? 4
2D LIFT CURVE SLOPE (/RAD) = ? 5.73
TIP LOSS FACTOR = ? 0.97
BLADE TWIST (DEG) = ? -8
EQUIVALENT FLAT PLATE AREA (M**2 OR FT**2) = ? 17
LOCKS NUMBER = ? 15
DRAG POLAR CO-EFFICIENT (DEL0) = ? 0.0087
DELTA1 (/RAD) = ? -0.0216
DELTA2 (/RAD**2) = ? 0.4
AIR DENSITY (KG/M**3 OR SLUG/FT**3) = ? 0.001327
IS RANGE/ENDURANCE REQUIRED ? (Y OR N) : N
IS A SPEED-POWER POLAR REQUIRED ? (Y OR N) : N
IS POWER REQUIRED FOR A GIVEN CLIMB ANGLE
i.e. RATE OF CLIMB (Y OR N) ? Y
VELOCITY IN KNOTS ? : 92
WHAT IS CLIMB ANGLE - GAMMA = ASIN(VV/V) - (DEG) ? : 0.615

STOP

END OF EXECUTION
CPU TIME: 1.38  ELAPSED TIME: 53.96
EXIT
```

```
.TY POLAR.OUT
SERVIVE CEILING ESTIMATE

 ALL UP WEIGHT =       9604.0 LB
 AIR DENSITY =   0.001327 SLUG/FT**3
 TIP SPEED =   717.2 FT/S
 ROTOR RADIUS =   21.0 FT
 ROTOR CHORD =   1.3 FT
 NUMBER OF ROTOR BLADES =   4.0
 LIFT CURVE SLOPE =   5.73 /RAD
 TIP LOSS FACTOR =   0.97
 BLADE TWIST =   -8.0 DEG
 EQUIVALENT FLAT PLATE AREA =   17.0 FT**2
 LOCKS NUMBER =   15.0
 DRAG POLAR CO-EFFICIENT (DELTA 0) =   0.0087
 DELTA 1 = -0.0216 /RAD
 DELTA 2 =   0.4000 /RAD**2
 AIRSPEED =   92.00 KNOTS

 ADVANCE RATIO(MU) =   0.214
 INDUCED VELOCITY(NU) =   16.49 FT/S
 INFLOW RATIO(LAMBDA) =   -0.0541
 FLAT PLATE DRAG =     272.0 LB
 THRUST =    9610.8 LB
 THRUST CO-EFFICIENT (CT) =   0.01016
 BLADE SOLIDITY =   0.0788
 COLLECTIVE (THETA 0) =   18.1 DEG
 CONING ANGLE (a0) =   13.6 DEG
 LONGITUDINAL FLAPPING ANGLE (a1) =   6.0 DEG
 LATERAL FLAPPING ANGLE (b1) =   3.9 DEG
 DISC ANGLE OF ATTACK =   -8.2 DEG
 INDUCED POWER =    288.2 HP
 PARASITE POWER =    76.8 HP
 PROFILE POWER =    196.2 HP
 TOTAL POWER =   561.1 HP
 REQUIRED/AVAILABLE SHAFT POWER =    590.2 HP
 CLIMB POWER =    29.1 HP
 RATE OF CLIMB =    100.0 FT/MIN
 CLIMB ANGLE  =   0.6 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =   11.2 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&TIP) =   13.8 DEG
 LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =   14.7 DEG
 RADIAL POINT OF UT=0.4 (@270DEG) =   0.614
```

The relevant parameters for this case are:

P_i = 288.2 Hp, P_p = 76.8 Hp, P_c = 29.1 Hp,

$P_{o_{calc}}$ = 196.2 Hp, $\alpha_{(1,270)}$ = 13.8°

$$\therefore P_o = 196.2\ (1 + \frac{13.8 - 12}{4}) = 284.5\ \text{Hp}$$

thus

$$P_{req} = P_i + P_p + P_c + P_o$$

$$= 679 \text{ Hp.}$$

Available shaft power is given by:

$$P_{avail} = 1275\ (1.13 \times 0.001327/0.002259 - 0.13)$$

$$= 681 \text{ Hp.}$$

Therefore, service ceiling is approximately 16,900 ft.

4.4.3 Hover ceiling Out of Ground Effect

As for estimating the other ceilings, the program must be run several times at different altitudes.

A run of 'POLAR' is shown below, which corresponds to an altitude of 3,400 ft, and is the result after several iterations.

```
.RU POLAR

TITLE (TWO LINES OF UP TO 60 CHARACTERS)
: BLADE ELEMENT METHOD - IMPERIAL UNITS
HOVER CEILING O.G.E.

ARE UNITS IN IMPERIAL OR METRIC (I OR M) ? I

ALL UP WEIGHT (N OR LB) = ? 9604

ROTOR TIP SPEED (M/S OR FT/S) = ? 717.2

ROTOR RADIUS (M OR FT) = ? 21

ROTOR BLADE CHORD (M OR FT) = ? 1.3

NUMBER OF ROTOR BLADES = ? 4

2D LIFT CURVE SLOPE (/RAD) = ? 5.73

TIP LOSS FACTOR = ? 0.97

BLADE TWIST (DEG) = ? -8

EQUIVALENT FLAT PLATE AREA (M**2 OR FT**2) = ? 17

LOCKS NUMBER = ? 15

DRAG POLAR CO-EFFICIENT (DEL0) = ? 0.0087

DELTA1 (/RAD) = ? -0.0216

DELTA2 (/RAD**2) = ? 0.4

AIR DENSITY (KG/M**3 OR SLUG/FT**3) = ? 0.002041

IS RANGE/ENDURANCE REQUIRED ? (Y OR N) : N

IS A SPEED-POWER POLAR REQUIRED ? (Y OR N) : N

IS POWER REQUIRED FOR A GIVEN CLIMB ANGLE
i.e. RATE OF CLIMB (Y OR N) ? Y

VELOCITY IN KNOTS ? : 0

WHAT IS CLIMB ANGLE - GAMMA = ASIN(VV/V) - (DEG) ? : 0

STOP

END OF EXECUTION
CPU TIME: 1.15  ELAPSED TIME: 37.68
EXIT

.
```

```
.TY POLAR.OUT
BLADE ELEMENT METHOD - IMPERIAL UNITS
HOVER CEILING O.G.E.
 ALL UP WEIGHT =      9604.0 LB
 AIR DENSITY =  0.002041 SLUG/FT**3
 TIP SPEED =  717.2 FT/S
 ROTOR RADIUS =  21.0 FT
 ROTOR CHORD =  1.3 FT
 NUMBER OF ROTOR BLADES =  4.0
 LIFT CURVE SLOPE =  5.73 /RAD
 TIP LOSS FACTOR =  0.97
 BLADE TWIST =  -8.0 DEG
 EQUIVALENT FLAT PLATE AREA =  17.0 FT**2
 LOCKS NUMBER =  15.0
 DRAG POLAR CO-EFFICIENT (DELTA 0) =  0.0087
 DELTA 1 = -0.0216 /RAD
 DELTA 2 =  0.4000 /RAD**2
 AIRSPEED =   0.00 KNOTS

 ADVANCE RATIO(MU) =  0.000
 INDUCED VELOCITY(NU) =  53.56 FT/S
 INFLOW RATIO(LAMBDA) =  -0.0747
 FLAT PLATE DRAG =      0.0 LB
 THRUST =   9604.0 LB
 THRUST CO-EFFICIENT (CT) =  0.00660
 BLADE SOLIDITY =  0.0788
 COLLECTIVE (THETA 0) =  17.9 DEG
 CONING ANGLE (a0) =   9.6 DEG
 LONGITUDINAL FLAPPING ANGLE (a1) =   0.0 DEG
 LATERAL FLAPPING ANGLE (b1) =   0.0 DEG
 DISC ANGLE OF ATTACK =   0.0 DEG
 INDUCED POWER =   935.2 HP
 PARASITE POWER =     0.0 HP
 PROFILE POWER =   204.3 HP
 TOTAL POWER =  1139.4 HP
 REQUIRED/AVAILABLE SHAFT POWER =  1139.4 HP
 CLIMB POWER =      0.0 HP
 RATE OF CLIMB =      0.0 FT/MIN
 CLIMB ANGLE  =   0.0 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =   4.1 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&TIP) =   5.6 DEG
 LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =   4.1 DEG
 RADIAL POINT OF UT=0.4 (@270DEG) =  0.400
```

Available shaft power is given by:

$$P_{avail} = 1275\ (1.13 \times 0.002041/0.002259 - 0.13)$$

$$= 1139\ \text{Hp.}$$

Thus hover ceiling O.G.E. is approximately 3400 ft.

4.4.4. Hover ceiling In Ground Effect

A ground effect relationship (ref. 8) is given as:

$$\frac{T}{T_\infty} = \frac{1}{1 - \frac{\sigma a|\lambda|}{4C_T} \frac{\left(\frac{R}{4Z}\right)^2}{1+(\mu/\lambda)^2}}$$

For example, taking z = R/2 and using the hover O.G.E. case as a basis:

$$\frac{T}{T_\infty} = \frac{1}{1 - \frac{0.0788 \times 5.73 \times 0.0747}{4 \times 0.00660} \frac{(1/2)^2}{1}}$$

$$= 1.4693.$$

A *helicopter of weight* 9604 lbs *hovering I.G.E.* is therefore equivalent to a helicopter of weight 9604/1.4693 = 6536 lbs hovering O.G.E.

'POLAR' is run as for §4.4.3 at this equivalent weight for an altitude of 14000 ft, and is the final result after several iterations.

```
.TY POLAR.OUT
BLADE ELEMENT METHOD - IMPERIAL UNITS
HOVER CEILING I.G.E.
 ALL UP WEIGHT =       6536.0 LB
 AIR DENSITY =  0.001461 SLUG/FT**3
 TIP SPEED =  717.2 FT/S
 ROTOR RADIUS =  21.0 FT
 ROTOR CHORD =  1.3 FT
 NUMBER OF ROTOR BLADES =  4.0
 LIFT CURVE SLOPE =  5.73 /RAD
 TIP LOSS FACTOR =  0.97
 BLADE TWIST =  -8.0 DEG
 EQUIVALENT FLAT PLATE AREA =  17.0 FT**2
 LOCKS NUMBER =  15.0
 DRAG POLAR CO-EFFICIENT (DELTA 0) =  0.0087
 DELTA 1 = -0.0216 /RAD
 DELTA 2 =  0.4000 /RAD**2
 AIRSPEED =   0.00 KNOTS

 ADVANCE RATIO(MU) =  0.000
 INDUCED VELOCITY(NU) =  52.22 FT/S
 INFLOW RATIO(LAMBDA) =  -0.0728
 FLAT PLATE DRAG =       0.0 LB
 THRUST =   6536.0 LB
 THRUST CO-EFFICIENT (CT) =  0.00628
 BLADE SOLIDITY =  0.0788
 COLLECTIVE (THETA 0) =  17.5 DEG
 CONING ANGLE (a0) =   9.1 DEG
 LONGITUDINAL FLAPPING ANGLE (a1) =   0.0 DEG
 LATERAL FLAPPING ANGLE (b1) =   0.0 DEG
 DISC ANGLE OF ATTACK =   0.0 DEG
 INDUCED POWER =   620.6 HP
 PARASITE POWER =      0.0 HP
 PROFILE POWER =   142.1 HP
 TOTAL POWER =   762.7 HP
 REQUIRED/AVAILABLE SHAFT POWER =   762.7 HP
 CLIMB POWER =     0.0 HP
 RATE OF CLIMB =     0.0 FT/MIN
 CLIMB ANGLE  =   0.0 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =   3.9 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&TIP) =   5.3 DEG
 LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =   3.9 DEG
 RADIAL POINT OF UT=0.4 (@270DEG) =  0.400
```

Here P_{req} = 763 Hp.

At 14,000 ft,

$$P_{avail} = 1275\ (1.13 \times 0.001461/0.002259 - 0.13)$$

$$= 766 \text{ Hp.}$$

Thus the Hover Ceiling I.G.E. is approximately 14,000 ft.

It should be noted that in some situations, a helicopter can hover I.G.E., but has insufficient power to move forward without decending. A check to see if this occurs can be made in the following manner:

(1) Take a case for low μ (say 0.02) in level flight at the hover O.G.E. ceiling.

(2) Calculate the I.G.E. thrust ratio.

(3) Calculate the equivalent O.G.E. weight.

(4) Run 'POLAR' for that μ at the hover I.G.E. ceiling.

(5) If the power required is greater than the power available, then the helicopter can hover but cannot move forward withcut descending.

(6) Repeat steps 1 → 5 over a range of μ, if required.

An example showing the results of this procedure is set out below:

```
.TY POLAR.OUT
BLADE ELEMENT METHOD - IMPERIAL UNITS
MU = 0.02, O.G.E.
 ALL UP WEIGHT =       9604.0 LB
 AIR DENSITY =  0.002041 SLUG/FT**3
 TIP SPEED =  717.2 FT/S
 ROTOR RADIUS =  21.0 FT
 ROTOR CHORD =  1.3 FT
 NUMBER OF ROTOR BLADES =  4.0
 LIFT CURVE SLOPE =  5.73 /RAD
 TIP LOSS FACTOR =  0.97
 BLADE TWIST =  -8.0 DEG
 EQUIVALENT FLAT PLATE AREA =  17.0 FT**2
 LOCKS NUMBER =  15.0
 DRAG POLAR CO-EFFICIENT (DELTA 0) =  0.0087
 DELTA 1 = -0.0216 /RAD
 DELTA 2 =  0.4000 /RAD**2
 AIRSPEED =   8.50 KNOTS

 ADVANCE RATIO(MU) =  0.020
 INDUCED VELOCITY(NU) =  50.17 FT/S
 INFLOW RATIO(LAMBDA) =  -0.0701
 FLAT PLATE DRAG =      3.6 LB
 THRUST =   9604.0 LB
 THRUST CO-EFFICIENT (CT) =  0.00660
 BLADE SOLIDITY =  0.0788
 COLLECTIVE (THETA 0) =  17.5 DEG
 CONING ANGLE (a0) =   9.6 DEG
 LONGITUDINAL FLAPPING ANGLE (a1) =   0.5 DEG
 LATERAL FLAPPING ANGLE (b1) =   0.3 DEG
 DISC ANGLE OF ATTACK =  -0.5 DEG
 INDUCED POWER =   876.1 HP
 PARASITE POWER =      0.1 HP
 PROFILE POWER =   202.9 HP
 TOTAL POWER =  1079.1 HP
 REQUIRED/AVAILABLE SHAFT POWER =  1079.1 HP
 CLIMB POWER =      0.0 HP
 RATE OF CLIMB =      0.0 FT/MIN
 CLIMB ANGLE  =   0.0 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =   4.3 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&TIP) =    5.9 DEG
 LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =   4.7 DEG
 RADIAL POINT OF UT=0.4 (@270DEG) =  0.420
```

Calculation of ground effect factor gives:

$$\frac{T}{T_\infty} = \frac{1}{1 - \dfrac{0.0788 \times 5.73 \times 0.0701}{4 \times 0.0066} \quad \dfrac{(1/2)^2}{1+(0.02/0.0701)^2}} = 1.3835$$

Thus equivalent O.G.E. weight is 9604/1.3835 = 6942 lbs.

```
.TY POLAR.OUT
BLADE ELEMENT METHOD - IMPERIAL UNITS
MU = 0.02, I.G.E.
 ALL UP WEIGHT =       6942.0 LB
 AIR DENSITY =   0.001461 SLUG/FT**3
 TIP SPEED =   717.2 FT/S
 ROTOR RADIUS =   21.0 FT
 ROTOR CHORD =   1.3 FT
 NUMBER OF ROTOR BLADES =   4.0
 LIFT CURVE SLOPE =   5.73 /RAD
 TIP LOSS FACTOR =   0.97
 BLADE TWIST =   -8.0 DEG
 EQUIVALENT FLAT PLATE AREA =   17.0 FT**2
 LOCKS NUMBER =   15.0
 DRAG POLAR CO-EFFICIENT (DELTA 0) =   0.0087
 DELTA 1 = -0.0216 /RAD
 DELTA 2 =  0.4000 /RAD**2
 AIRSPEED =    8.50 KNOTS

 ADVANCE RATIO(MU) =   0.020
 INDUCED VELOCITY(NU) =   50.43 FT/S
 INFLOW RATIO(LAMBDA) =   -0.0705
 FLAT PLATE DRAG =        2.6 LB
 THRUST =    6942.0 LB
 THRUST CO-EFFICIENT (CT) =   0.00667
 BLADE SOLIDITY =   0.0788
 COLLECTIVE (THETA 0) =    17.6 DEG
 CONING ANGLE (a0) =    9.6 DEG
 LONGITUDINAL FLAPPING ANGLE (a1) =    0.5 DEG
 LATERAL FLAPPING ANGLE (b1) =    0.3 DEG
 DISC ANGLE OF ATTACK =   -0.5 DEG
 INDUCED POWER =    636.6 HP
 PARASITE POWER =       0.1 HP
 PROFILE POWER =    146.0 HP
 TOTAL POWER =    782.7 HP
 REQUIRED/AVAILABLE SHAFT POWER =    782.7 HP
 CLIMB POWER =       0.0 HP
 RATE OF CLIMB =       0.0 FT/MIN
 CLIMB ANGLE  =    0.0 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =    4.3 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&TIP) =    5.9 DEG
 LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =    4.7 DEG
 RADIAL POINT OF UT=0.4 (@270DEG) =   0.420
```

Now P_{req} is 783 Hp, but available power at 14,000 ft is 766 Hp (found previously).

Thus if helicopter moves forward, it will descend.

5. COMPARISON OF 'POLAR' AND OTHER METHODS OF PERFORMANCE ESTIMATION

As mentioned in Section 1, the methods of performance estimation available prior to 'POLAR' were:

1. Program 'ENERGY' (see Appendix 4), derived from 'MCEP', and
2. Performance Charts of References 1 and 2.

This section compares results obtained for the hypothetical helicopter of Section 4, using 'POLAR' and the above-mentioned methods.

5.1 'POLAR' and 'ENERGY'

Figure 4 shows an overlay plot of speed against power, with a breakdown of power components, using 'POLAR' and 'ENERGY'.

As can be seen from this figure, agreement of the power components is generally good. The main discrepancy between 'ENERGY' and 'POLAR' occurs at high speed. This is partially due to 'POLAR' predicting a lower profile power, but mostly due to compressibility power calculated by 'ENERGY'. 'POLAR' does not take compressibility effects into account.

5.2 'POLAR' and Performance Charts

Figures 5, 6 and 7 show power-speed polars obtained using 'POLAR', together with points obtained from the performance charts of references 1 and 2, for rotor blade twists of 0, 8 and 16 degrees respectively.

5.2.1 'POLAR' and Performance Charts of Gessow and Tapscott

From Figure 5, it can be seen that agreement is good between 'POLAR' and the Reference 2 charts, except at low speed. This is due to the fact that 'POLAR' applies a semi-empirical factor to the induced velocity (for $\mu \leq 0.14$) whereas the charts use only the momentum theory value (see §2.2). Thus the charts always underestimate the induced velocity (induced power) at low speed.

Figures 6 and 7 also show good agreement, except at low speed where the induced power is underestimated by the charts, as expected. For both cases (8 and 16° twist), at high speed 'POLAR' underestimates the profile power because 'POLAR' ignores the high order terms in μ and θ_1, in the profile power expression (see §2.5). Thus at high speeds (μ), neglect of these terms comes into effect. For example, at $\mu = 0.3$, these terms cause a 13% difference in profile power for 8 and 16 degrees twist, but this represents only a 5% difference in total power.

5.2.2 'POLAR' and Performance Charts of Tanner

The performance charts of Reference 1 are applicable only to hovering helicopters and aircraft flying at $\mu \geq 0.25$ for 8° rotor twist and $\mu \geq 0.3$ for 0° twist.

Figure 5 shows poor agreement for the total power. In the hover case this is due to the semi-empirical factor applied at low speed by 'POLAR' (as in §5.2.1). At $\mu = 0.3$, the charts of Reference 1 indicate that the rotor is stalled. This discrepancy is thus probably due to Tanner's inclusion of compressibility and stall effects, which are neglected in the case of 'POLAR'.

Figure 6 again shows poor agreement at hover, for the reason mentioned previously. However, at $\mu = 0.25$ and $\mu = 0.3$ we can see excellent agreement. In this case the rotor is not stalled, thus verifying that the difference in Figure 5 is due to Tanner's stall effect.

6. CONCLUDING REMARKS

Program 'POLAR' has been developed to give a wide range of performance calculations within the operational boundaries of most conventional helicopters.

'POLAR' allows rapid estimation and is more generally applicable than existing performance charts.

Being based on blade element theory, 'POLAR' overcomes the deficiencies of the energy balance method (program 'ENERGY') in that

blade twist is taken into account and blade stall is indicated by calculation of blade angles of attack at critical locations on the rotor disk.

Since 'POLAR' does not take compressibility and stall effects into account, it is limited to $\mu \leq 0.3$.

REFERENCES

[1] Watson H. Tanner: "Charts for Estimating Rotary Wing Performance in Hover and at High Forward Speeds", NASA CR-114, November 1964.

[2] Alfred Gessow and Robert J. Tapscott: "Charts for Estimating Performance of High Performance Helicopters", NACA Report 1266, 1956.

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[4] Wood, T. and Waak, T.: "Improved Maneuver Criteria Evaluation Program", USARTL-TR-79-20, November 1979.

[5] Alfred Gessow and Almer D. Crim: "An Extension of Lifting Rotor Theory to Cover Operation at Large Angles of Attack and High Inflow Conditions", NACA TN2665, April 1952.

[6] Bramwell, A.R.S.: "Helicopter Dynamics", Edward Arnold (Publishers) Ltd., 1976.

[7] Wheatly, J.B.: "An Aerodynamic Analysis of the Autogiro Rotor with a Comparison Between Calculated and Experimental Results", NACA Rep. 487, 1934.

[8] Johnson, W.: "Helicopter Theory", Princeton University Press, 1980.

[9] Stepniewski, W.Z.: "Comparative Study of Soviet vs Western Helicopters Part I", NACA CR3579, March 1983.

[10] Gustafson, F.B. and Gessow, A.: "Effect of Blade Stalling on the Efficiency of a Helicopter Rotor As Measured in Flight", NACA TN 1250, April 1947.

[11] Nankivell, P.G. and Gilbert, N.E.: "A General Purpose Output Program for Use in Simulation", ARL/Aero Note 367, December 1976.

[12] Hall, P.H.: "A set of Flight Dynamic Equations for Aircraft Simulation", ARL/Aero Tech Memo 339.

[13] Gessow and Myers: "Aerodynamics of the Helicopter", Macmillan, 1952.

APPENDIX 1 - USING PROGRAM 'POLAR'

A1.1 Loading 'POLAR' on the DEC-10

For ARL users, a description is shown below of how to load program 'POLAR' on the DEC-10 computer. 'POLAR' is also available on the VAX-11 computer and is loaded in a similar fashion.

```
.PLE MX D/039
MESSAGE TRANSMITTED at 13:29

.
;;OPR: - D/039 MX DTA0
.AS DTA0
DTA000 assigned

.DIR DTA0:

Tape ID: D/039
Free: 428 blks, 13 files
POLAR .CCL    1  20-Apr-83
POLAR .FOR   60   7-Sep-83
DING  .REL    1  14-Apr-83
ENERGY.CCL    1  20-Apr-83
ENERGY.FOR   32   7-Sep-83
BLADE .FOR   48   [illegible]-Jun-83
TSUB81.REL   10  30-Mar-83
CPU   .REL    1   9-Nov-79
ATMOS .FOR   15  23-Jun-83

.COPY/X=DTA0:POLAR.*,TSUB81.*,CPU.*,DING.*
.COM POLAR
FORTRAN: POLAR
POLAR
ELMENT
LABDA
TAN
IMPLIC
COEFF
POWER
ALPH
ROC
CLIMB
RNGEND

.TY POLAR.CCL
POLAR,TSUB81,CPU,DING
POLAR/SAV=
/GO

.R LINK

*@POLAR

EXIT

.DIR POLAR.EXE

POLAR    EXE    60   <057>    7-SEP-83    DSKD:   [1031,1063]
.
```

A1.2 Output Files

When running 'POLAR', the results are output onto one of two files. If performance is required at one particular airspeed, or range/endurance are desired, the results are output onto file 'POLAR.OUT'. If a speed-power polar is required, the results are output onto file 'POLAR.DAT', which is then input to program 'TRANS' for presentation in column or graphical form with velocity as the independent variable. An example of this procedure is given in Appendix 2.

A1.3 Renaming Files

A file renaming feature is included in 'POLAR' to allow up to five consecutive runs to be made without overwriting the results obtained from a previous run. As stated above, results for each run are always output to files POLAR.OUT or POLAR.DAT If a second run is made, the preceeding POLAR.OUT or POLAR.DAT will automatically be renamed POL1.OUT or POL1.DAT. This process will continue up to POL4.OUT or POL4.DAT.

A1.4 Special Cases

It was found that for some flight conditions, when calculating rate of climb for a given shaft power and airspeed, convergence would not occur unless the climb angle, γ_c, could be roughly estimated initially. Therefore, 'POLAR' calculates the power required in level flight (P_{level}) and the power required in a vertical climb (P_{vert}) and uses a linear interpolation to initially approximate γ_c.

i.e. $$\gamma_c = (P_{avail} - P_{level})/(P_{vert} - P_{avail})\ 90°$$

In some cases when the rate of climb is required for a given airspeed and shaft power, the power may not be sufficient to sustain a climb and therefore the helicopter descends. 'POLAR' outputs a message to this effect which is demonstrated below.

```
.RU POLAR

TITLE (TWO LINES OF UP TO 60 CHARACTERS)
: BLADE ELEMENT METHOD - IMPERIAL UNITS
DESCENDING FLIGHT

ARE UNITS IN IMPERIAL OR METRIC (I OR M) ? I

ALL UP WEIGHT (N OR LB) = ? 9604

ROTOR TIP SPEED (M/S OR FT/S) = ? 717.2

ROTOR RADIUS (M OR FT) = ? 21

ROTOR BLADE CHORD (M OR FT) = ? 1.3

NUMBER OF ROTOR BLADES = ? 4

2D LIFT CURVE SLOPE (/RAD) = ? 5.73

TIP LOSS FACTOR = ? 0.97

BLADE TWIST (DEG) = ? -8

EQUIVALENT FLAT PLATE AREA (M**2 OR FT**2) = ? 17

LOCKS NUMBER = ? 15

DRAG POLAR CO-EFFICIENT (DEL0) = ? 0.0087

DELTA1 (/RAD) = ? -0.0216

DELTA2 (/RAD**2) = ? 0.4

AIR DENSITY (KG/M**3 OR SLUG/FT**3) = ? 0.002259

IS RANGE/ENDURANCE REQUIRED ? (Y OR N) : N

IS A SPEED-POWER POLAR REQUIRED ? (Y OR N) : N

IS POWER REQUIRED FOR A GIVEN CLIMB ANGLE
i.e. RATE OF CLIMB (Y OR N) ? N

VELOCITY IN KNOTS ? : 12

WHAT IS POWER AVAILABLE AT MAIN ROTOR SHAFT (HP) ? : 1000

STOP

END OF EXECUTION
CPU TIME: 1.27  ELAPSED TIME: 36.78
EXIT

.
```

```
.TY POLAR.OUT
 HELICOPTER HAS INSUFFICIENT POWER FOR LEVEL FLIGHT
 i.e. AIRCRAFT IS DESCENDING

BLADE ELEMENT METHOD - IMPERIAL UNITS
DESCENDING FLIGHT
 ALL UP WEIGHT =      9604.0 LB
 AIR DENSITY =  0.002259 SLUG/FT**3
 TIP SPEED =  717.2 FT/S
 ROTOR RADIUS =  21.0 FT
 ROTOR CHORD =  1.3 FT
 NUMBER OF ROTOR BLADES =  4.0
 LIFT CURVE SLOPE =  5.73 /RAD
 TIP LOSS FACTOR =  0.97
 BLADE TWIST =  -8.0 DEG
 EQUIVALENT FLAT PLATE AREA =  17.0 FT**2
 LOCKS NUMBER =  15.0
 DRAG POLAR CO-EFFICIENT (DELTA 0) =  0.0087
 DELTA 1 = -0.0216 /RAD
 DELTA 2 =  0.4000 /RAD**2
 AIRSPEED =  12.00 KNOTS

 ADVANCE RATIO(MU) =  0.028
 INDUCED VELOCITY(NU) =  45.48 FT/S
 INFLOW RATIO(LAMBDA) =  -0.0632
 FLAT PLATE DRAG =      7.9 LB
 THRUST =  9603.9 LB
 THRUST CO-EFFICIENT (CT) =  0.00597
 BLADE SOLIDITY =  0.0788
 COLLECTIVE (THETA 0) =  16.4 DEG
 CONING ANGLE (a0) =  8.6 DEG
 LONGITUDINAL FLAPPING ANGLE (a1) =  0.6 DEG
 LATERAL FLAPPING ANGLE (b1) =  0.3 DEG
 DISC ANGLE OF ATTACK =  0.4 DEG
 INDUCED POWER =  794.2 HP
 PARASITE POWER =     0.3 HP
 PROFILE POWER =  211.9 HP
 TOTAL POWER =  1006.4 HP
 REQUIRED/AVAILABLE SHAFT POWER =  1000.0 HP
 CLIMB POWER =    -6.4 HP
 RATE OF CLIMB =   -22.1 FT/MIN
 CLIMB ANGLE  =  -1.0 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =   4.1 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&TIP) =   5.2 DEG
 LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =   4.6 DEG
 RADIAL POINT OF UT=0.4 (@270DEG) =  0.428
```

A further case to be considered is when the rate of climb is required for a given airspeed and shaft power, and the available power is greater than that required for vertical flight. 'POLAR' then outputs a message stating the excess power available and then gives results for a vertical climb at the specified airspeed. In the example below, the inputs are identical to the previous example except that the power available is now 1275 Hp. Only the output file is given.

```
.TY POLAR.OUT
 HELICOPTER HAS   12.9 HP MORE THAN IS
 REQUIRED FOR A VERTICAL CLIMB. RESULTS BELOW
 ARE THOSE FOR A VERTICAL CLIMB.

BLADE ELEMENT METHOD - IMPERIAL UNITS
EXCESS POWER DEMONSTRATION
 ALL UP WEIGHT =      9604.0 LB
 AIR DENSITY =  0.002259 SLUG/FT**3
 TIP SPEED =  717.2 FT/S
 ROTOR RADIUS =  21.0 FT
 ROTOR CHORD =  1.3 FT
 NUMBER OF ROTOR BLADES =  4.0
 LIFT CURVE SLOPE =  5.73 /RAD
 TIP LOSS FACTOR =  0.97
 BLADE TWIST =  -8.0 DEG
 EQUIVALENT FLAT PLATE AREA =  17.0 FT**2
 LOCKS NUMBER =  15.0
 DRAG POLAR CO-EFFICIENT (DELTA 0) =   0.0087
 DELTA 1 = -0.0216 /RAD
 DELTA 2 =  0.4000 /RAD**2
 AIRSPEED =  12.00 KNOTS

 ADVANCE RATIO(MU) =   0.000
 INDUCED VELOCITY(NU) =   39.44 FT/S
 INFLOW RATIO(LAMBDA) =   -0.0832
 FLAT PLATE DRAG =        7.9 LB
 THRUST =    9612.4 LB
 THRUST CO-EFFICIENT (CT) =  0.00597
 BLADE SOLIDITY =  0.0788
 COLLECTIVE (THETA 0) =   18.1 DEG
 CONING ANGLE (a0) =    8.9 DEG
 LONGITUDINAL FLAPPING ANGLE (a1) =   0.0 DEG
 LATERAL FLAPPING ANGLE (b1) =   0.0 DEG
 DISC ANGLE OF ATTACK = -90.0 DEG
 INDUCED POWER =    689.3 HP
 PARASITE POWER =      0.3 HP
 PROFILE POWER =    218.9 HP
 TOTAL POWER =    908.5 HP
 REQUIRED/AVAILABLE SHAFT POWER =  1275.0 HP
 CLIMB POWER =    353.7 HP
 RATE OF CLIMB =   1215.2 FT/MIN
 CLIMB ANGLE  =   90.0 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&40% RADIUS ) =    3.1 DEG
 LOCAL ANGLE OF ATTACK(@270DEG&TIP) =     5.3 DEG
 LOCAL ANGLE OF ATTACK (@270DEG&PT.OF UT=0.4) =    3.1 DEG
 RADIAL POINT OF UT=0.4 (@270DEG) =   0.400
```

APPENDIX 2 - USING PROGRAM 'TRANS'

Program 'TRANS' (Ref. 11) was created to plot and tabulate parameters from a '.DAT' file, with time as the independent variable. When using input files created by 'POLAR' and 'ENERGY' (Appendix 4), the independent variable is velocity. Thus the time limit command of 'TRANS', TIM, now refers to the range and intervals of velocity.

A2.1 Obtaining a Speed-Power Polar

The example below shows how to run 'TRANS' in order to create a plot of total power against velocity i.e. a speed-power polar (Fig. 3). The input file used was created using 'POLAR' in §4.1.

```
.RU PUB:TRANS

[TRANS version date 12-NOV-80]

I/P FILENAME = POLAR

BLADE ELEMENT METHOD
  POWER - SPEED POLAR

I/P FILE RECORDED ON 28-Sep-83 AT 09:09

INTEGN INT = 0.0000E+00; RUN CPU TIME =  4.02 SEC.

TIME FROM 0.0000E+00 TO 1.5000E+02 IN STEPS OF 2.0000E+00

(Now interpreted as velocity range from 0 to 150 kts in steps of 2 kts)

*SPA

IS SPACING BETWEEN PLOTS REQRD : N

*SCA

BLK NO. -1 DENOTES INDEP VARIABLE

ARE PLOT SCALE LIMITS TO BE READ FROM DSK : N

IS TTY LISTING OF LIMITS REQRD : N

ARE MODIFICATIONS REQRD : Y

BLK, LOWER, UPPER
1,0,1500

*PLS

STRIP PLOTS :

BLKS
1

TO SPECIFY NO. OF X UNITS/INCH, TYPE 0 FOR X

LENGTH OF AXES IN INCHES; X, Y = 0,6

MIN X, NO. OF X UNITS/INCH = 0,30

ARE SYMBOLS REQRD FOR PLOTS : N

LINE KEY (0 GIVES DEFAULT) = 0

*GOE

** RUNNING **

*EXI

END OF EXECUTION
CPU TIME: 3.20  ELAPSED TIME: 1:7.54
EXIT
.
```

A2.2 Tabulated Printout

The example below shows how 'TRANS' is run to create a file 'POLAR.COL', containing tabulated data over a range of velocities.

```
.RU PUB:TRANS

[TRANS version date 12-NOV-80]

I/P FILENAME = POLAR

BLADE ELEMENT METHOD
  POWER - SPEED POLAR

I/P FILE RECORDED ON 28-Sep-83 AT 09:10

INTEGN INT = 0.0000E+00; RUN CPU TIME =  4.02 SEC.

TIME FROM 0.0000E+00 TO 1.5000E+02 IN STEPS OF 2.0000E+00

*TIM

TIME PARAMS; LOWER, UPPER, INTERVAL = 60,80,2

*PRC

PRINTING IN COLUMNS :

BLKS
A

IS O/P TO TTY REQRD : N

*GOE

** RUNNING **

*TIM

TIME PARAMS; LOWER, UPPER, INTERVAL = 100,120,2

*PRC

PRINTING IN COLUMNS :

BLKS
A

IS O/P TO TTY REQRD : N

*GOE

** RUNNING **

*EXI

END OF EXECUTION
CPU TIME: 3.62  ELAPSED TIME: 57.10
EXIT
.
```

1 BLADE ELEMENT METHOD
POWER - SPEED POLAR

RECORDED ON 28-Sep-83 AT 09:09 INTEGN INT = 0.0000E+00
+ RUN CPU TIME = 4.66 SEC.

NON-VARYING BLOCK VALUES

BLK NUMBER =	6	7	8	9	10	11	12	13	14	15
	BLADE SOLIDITY	MAX WEIGHT	DENSITY	TIP SPEED	RADIUS	CHORD	NUMBER OF BLADES	SMALL A DEG	TIP LOSS FACTOR	TWIST (DEG)
	7.8583E-02	4.2718E+04	1.1644E+00	2.1860E+02	6.4000E+00	3.9500E-01	4.0000E+00	5.7300E+00	9.7000E-01	-8.0000E+00

BLK NUMBER =	16	17	18	19	20	34	35	36
	F.P.AREA	LOCKS NUMBER	DELTA 0	DELTA 1	DELTA 2	BLK# 34	BLK# 35	BLK# 36
	1.5800E+00	1.5000E+01	8.7000E-03	-2.1600E-02	4.0000E-01	0.0000E+00	0.0000E+00	0.0000E+00

1 BLADE ELEMENT METHOD
POWER - SPEED POLAR

RECORDED ON 28-Sep-83 AT 09:09 INTEGN INT = 0.0000E+00
+ RUN CPU TIME = 4.66 SEC.

BLK NUMBER =	1	2	3	4	5	21	22	23	24	25
VELOCITY (KNOTS)	TOTAL HP	PROFILE HP	INDUCED HP	PARASITE HP	THRUST CO-EFF	MU	NU	F.P.DRAG	THETA 0 (DEG)	SMALL A0 (DEG)
6.0000E+01	5.1064E+02	2.1474E+02	2.5962E+02	3.6281E+01	5.9675E-03	1.4093E-01	4.5305E+00	8.7640E+02	1.3142E+01	7.9038E+00
6.2000E+01	5.0721E+02	2.1556E+02	2.5162E+02	4.0032E+01	5.9675E-03	1.4561E-01	4.3908E+00	9.3580E+02	1.3126E+01	7.8918E+00
6.4000E+01	5.0452E+02	2.1641E+02	2.4408E+02	4.4032E+01	5.9675E-03	1.5028E-01	4.2590E+00	9.9715E+02	1.3116E+01	7.8804E+00
6.6000E+01	5.0253E+02	2.1729E+02	2.3696E+02	4.8290E+01	5.9675E-03	1.5494E-01	4.1346E+00	1.0604E+03	1.3112E+01	7.8696E+00
6.8000E+01	5.0123E+02	2.1819E+02	2.3022E+02	5.2815E+01	5.9683E-03	1.5961E-01	4.0169E+00	1.1257E+03	1.3112E+01	7.8595E+00
7.0000E+01	5.0059E+02	2.1912E+02	2.2385E+02	5.7613E+01	5.9683E-03	1.6426E-01	3.9056E+00	1.1929E+03	1.3117E+01	7.8498E+00
7.2000E+01	5.0059E+02	2.2008E+02	2.1782E+02	6.2694E+01	5.9683E-03	1.6892E-01	3.8001E+00	1.2620E+03	1.3128E+01	7.8406E+00
7.4000E+01	5.0121E+02	2.2106E+02	2.1209E+02	6.8065E+01	5.9691E-03	1.7357E-01	3.7000E+00	1.3331E+03	1.3143E+01	7.8320E+00
7.6000E+01	5.0244E+02	2.2206E+02	2.0665E+02	7.3734E+01	5.9691E-03	1.7821E-01	3.6049E+00	1.4061E+03	1.3162E+01	7.8239E+00
7.8000E+01	5.0426E+02	2.2308E+02	2.0148E+02	7.9710E+01	5.9698E-03	1.8285E-01	3.5145E+00	1.4811E+03	1.3186E+01	7.8163E+00
8.0000E+01	5.0667E+02	2.2411E+02	1.9656E+02	8.6000E+01	5.9698E-03	1.8749E-01	3.4285E+00	1.5580E+03	1.3215E+01	7.8092E+00

1 BLADE ELEMENT METHOD
POWER - SPEED POLAR

RECORDED ON 28-Sep-83 AT 09:09 INTEGN INT = 0.0000E+00
+ RUN CPU TIME = 4.66 SEC.

BLK NUMBER =	26	27	28	29	30	31	32	33
VELOCITY (KNOTS)	SMALL A1 (DEG)	SMALL B1 (DEG)	ALPHA (DEG)	LAMBDA	LOCAL AOA @270(DEG)	LOCAL AOA @TIP(DEG)	LOCAL AOA AT UT=0.4	RADIAL PT. AT UT=0.4
6.0000E+01	2.3494E+00	1.5132E+00	-3.5247E+00	-2.9406E-02	7.0681E+00	5.9152E+00	7.7794E+00	5.4093E-01
6.2000E+01	2.4221E+00	1.5599E+00	-3.6770E+00	-2.9443E-02	7.1056E+00	5.9866E+00	7.8478E+00	5.4561E-01
6.4000E+01	2.4954E+00	1.6065E+00	-3.8326E+00	-2.9550E-02	7.1357E+00	6.0604E+00	7.9143E+00	5.5028E-01
6.6000E+01	2.5694E+00	1.6529E+00	-3.9915E+00	-2.9725E-02	7.1578E+00	6.1367E+00	7.9790E+00	5.5494E-01
6.8000E+01	2.6443E+00	1.6992E+00	-4.1537E+00	-2.9967E-02	7.1717E+00	6.2153E+00	8.0422E+00	5.5961E-01
7.0000E+01	2.7199E+00	1.7453E+00	-4.3194E+00	-3.0274E-02	7.1767E+00	6.2963E+00	8.1038E+00	5.6426E-01
7.2000E+01	2.7964E+00	1.7912E+00	-4.4886E+00	-3.0644E-02	7.1721E+00	6.3795E+00	8.1641E+00	5.6892E-01
7.4000E+01	2.8738E+00	1.8370E+00	-4.6613E+00	-3.1078E-02	7.1575E+00	6.4650E+00	8.2232E+00	5.7357E-01
7.6000E+01	2.9522E+00	1.8827E+00	-4.8375E+00	-3.1574E-02	7.1321E+00	6.5529E+00	8.2813E+00	5.7821E-01
7.8000E+01	3.0316E+00	1.9282E+00	-5.0174E+00	-3.2131E-02	7.0951E+00	6.6430E+00	8.3384E+00	5.8285E-01
8.0000E+01	3.1121E+00	1.9735E+00	-5.2009E+00	-3.2750E-02	7.0455E+00	6.7353E+00	8.3947E+00	5.8749E-01

1 BLADE ELEMENT METHOD
POWER - SPEED POLAR

RECORDED ON 28-Sep-83 AT 09:09 INTEGN INT = 0.0000E+00
+ RUN CPU TIME = 4.66 SEC.

NON-VARYING BLOCK VALUES

BLK NUMBER =	6	7	8	9	10	11	12	13	14	15
	BLADE SOLIDITY	MAX WEIGHT	DENSITY	TIP SPEED	RADIUS	CHORD	NUMBER OF BLADES	SMALL A DEG	TIP LOSS FACTOR	TWIST (DEG)
	7.8583E-02	4.2718E+04	1.1644E+00	2.1860E+02	6.4000E+00	3.9500E-01	4.0000E+00	5.7300E+00	9.7000E-01	-8.0000E+00

BLK NUMBER =	16	17	18	19	20	34	35	36
	F.P.AREA	LOCKS NUMBER	DELTA 0	DELTA 1	DELTA 2	BLK# 34	BLK# 35	BLK# 36
	1.5800E+00	1.5000E+01	8.7000E-03	-2.1600E-02	4.0000E-01	0.0000E+00	0.0000E+00	0.0000E+00

1 BLADE ELEMENT METHOD
POWER - SPEED POLAR

RECORDED ON 28-Sep-83 AT 09:09 INTEGN INT = 0.0000E+00
+ RUN CPU TIME = 4.66 SEC.

BLK NUMBER =	1	2	3	4	5	21	22	23	24	25
VELOCITY (KNOTS)	TOTAL HP	PROFILE HP	INDUCED HP	PARASITE HP	THRUST CO-EFF	MU	NU	F.P.FRAC	THETA 0 (DEG)	SMALL A0 (DEG)
1.0000E+02	5.6105E+02	2.3509E+02	1.5799E+02	1.6797E+02	5.9759E-03	2.3345E-01	2.7530E+00	2.4344E+03	1.3741E+01	7.7640E+00
1.0200E+02	5.6941E+02	2.3621E+02	1.5496E+02	1.7825E+02	5.9767E-03	2.3800E-01	2.6999E+00	2.5328E+03	1.3817E+01	7.7620E+00
1.0400E+02	5.7830E+02	2.3732E+02	1.5204E+02	1.8894E+02	5.9775E-03	2.4254E-01	2.6487E+00	2.6331E+03	1.3898E+01	7.7605E+00
1.0600E+02	5.8771E+02	2.3842E+02	1.4924E+02	2.0005E+02	5.9784E-03	2.4706E-01	2.5995E+00	2.7353E+03	1.3983E+01	7.7594E+00
1.0800E+02	5.9765E+02	2.3951E+02	1.4654E+02	2.1159E+02	5.9794E-03	2.5157E-01	2.5521E+00	2.8395E+03	1.4072E+01	7.7594E+00
1.1000E+02	6.0810E+02	2.4059E+02	1.4394E+02	2.2357E+02	5.9804E-03	2.5607E-01	2.5064E+00	2.9457E+03	1.4166E+01	7.7594E+00
1.1200E+02	6.1908E+02	2.4166E+02	1.4144E+02	2.3598E+02	5.9814E-03	2.6056E-01	2.4623E+00	3.0538E+03	1.4264E+01	7.7594E+00
1.1400E+02	6.3057E+02	2.4270E+02	1.3902E+02	2.4885E+02	5.9826E-03	2.6503E-01	2.4198E+00	3.1638E+03	1.4366E+01	7.7603E+00
1.1600E+02	6.4259E+02	2.4372E+02	1.3669E+02	2.6218E+02	5.9837E-03	2.6948E-01	2.3788E+00	3.2758E+03	1.4473E+01	7.7619E+00
1.1800E+02	6.5514E+02	2.4472E+02	1.3444E+02	2.7598E+02	5.9850E-03	2.7392E-01	2.3392E+00	3.3897E+03	1.4585E+01	7.7643E+00
1.2000E+02	6.6821E+02	2.4569E+02	1.3227E+02	2.9025E+02	5.9863E-03	2.7835E-01	2.3009E+00	3.5056E+03	1.4701E+01	7.7671E+00

1 BLADE ELEMENT METHOD
POWER - SPEED POLAR

RECORDED ON 28-Sep-83 AT 09:09 INTEGN INT = 0.0000E+00
+ RUN CPU TIME = 4.66 SEC.

BLK NUMBER =	26	27	28	29	30	31	32	33
VELOCITY (KNOTS)	SMALL A1 (DEG)	SMALL B1 (DEG)	ALPHA (DEG)	LAMBDA	LOCAL AOA @270(DEG)	LOCAL AOA @TIP(DEG)	LOCAL AOA AT UT=0.4	RADIAL PT. AT UT=0.4
1.0000E+02	3.9879E+00	2.4192E+00	-7.2496E+00	-4.2291E-02	5.5820E+00	7.7813E+00	8.9308E+00	6.3345E-01
1.0200E+02	4.0838E+00	2.4630E+00	-7.4769E+00	-4.3586E-02	5.3002E+00	7.8981E+00	8.9833E+00	6.3800E-01
1.0400E+02	4.1813E+00	2.5067E+00	-7.7085E+00	-4.4946E-02	4.9842E+00	8.0171E+00	9.0360E+00	6.4254E-01
1.0600E+02	4.2807E+00	2.5502E+00	-7.9444E+00	-4.6369E-02	4.6307E+00	8.1383E+00	9.0888E+00	6.4706E-01
1.0800E+02	4.3818E+00	2.5937E+00	-8.1848E+00	-4.7858E-02	4.2361E+00	8.2617E+00	9.1417E+00	6.5157E-01
1.1000E+02	4.4849E+00	2.6370E+00	-8.4295E+00	-4.9414E-02	3.7965E+00	8.3873E+00	9.1949E+00	6.5607E-01
1.1200E+02	4.5898E+00	2.6803E+00	-8.6788E+00	-5.1036E-02	3.3074E+00	8.5151E+00	9.2484E+00	6.6056E-01
1.1400E+02	4.6968E+00	2.7234E+00	-8.9325E+00	-5.2726E-02	2.7641E+00	8.6451E+00	9.3023E+00	6.6503E-01
1.1600E+02	4.8058E+00	2.7664E+00	-9.1909E+00	-5.4485E-02	2.1611E+00	8.7773E+00	9.3565E+00	6.6948E-01
1.1800E+02	4.9169E+00	2.8095E+00	-9.4539E+00	-5.6313E-02	1.4926E+00	8.9118E+00	9.4113E+00	6.7392E-01
1.2000E+02	5.0302E+00	2.8524E+00	-9.7215E+00	-5.8212E-02	7.5200E-01	9.0483E+00	9.4665E+00	6.7835E-01

APPENDIX 3 - USING PROGRAM 'ATMOS'

The atmospheric density, required for 'POLAR', and the speed of sound, required for 'ENERGY', may be found using the program 'ATMOS'. (Reference 12). For ARL users, 'ATMOS' is available on Dectape D/039 for use on the DEC-10 computer, and Magtape M/1012 for use on the VAX-11.

The following types of atmosphere may be considered by setting the KEYAIR flag appropriately.

KEYAIR = 1 : ICAO Standard Atmosphere

KEYAIR = 2 : ICAO Sea-level conditions at all times

KEYAIR = 3 : Off-Standard ICAO atmosphere

KEYAIR = 4 : ARDU Tropical atmosphere

KEYAIR = 5 : ARDU Sea-level conditions at all times

KEYAIR = 6 : Off-standard ARDU atmosphere

Examples of running 'ATMOS' are shown below, demonstrating its use for ISA+15°C conditions.

```
.RU ATMOS
SET ATMOSPHERIC FLAG, KEYAIR (1,2,3,4,5 OR 6): 3
SINGLE CALCULATION, OR TABLE (1 OR 2): 2
*STATE ALTITUDE (IN FEET): 20000
TEMPERATURE OF THE DAY, TDAY (IN DEG. C): 30
QNH OF THE DAY (IN MILLIBARS): 1013.25
HEIGHT OF THE AIRFIELD REFERENCE POINT, HAFR: 0
PAUSE
Type C to Continue, X to Exit, T to Trace.
*X

END OF EXECUTION
CPU TIME: 0.51  ELAPSED TIME: 1.26
EXIT
.
```

* 'ATMOS' calculates results in intervals of 1000ft up to stated altitude.

The results are stored in file 'ATMOS.OUT'.

Column 1 is altitude in feet

Column 2 is temperature in Kelvin

Column 3 is pressure in lbs/ft²

Column 4 is density in slug/ft³

Column 5 is speed of sound in ft/s.

```
.TY ATMOS.OUT
    0.0000000     303.1499939    2116.2199707         0.0022593    1145.1402588
 1000.0000000     301.1687927    2040.8562012         0.0021932    1141.3920898
 2000.0000000     299.1875916    1967.6798096         0.0021285    1137.6317139
 3000.0000000     297.2063904    1896.6431885         0.0020654    1133.8587646
 4000.0000000     295.2251892    1827.6977539         0.0020036    1130.0732422
 5000.0000000     293.2439880    1760.7960205         0.0019433    1126.2750244
 6000.0000000     291.2627869    1695.8903809         0.0018844    1122.4638672
 7000.0000000     289.2815857    1632.9365234         0.0018269    1118.6398926
 8000.0000000     287.3003845    1571.8883057         0.0017707    1114.8026123
 9000.0000000     285.3191833    1512.7016602         0.0017159    1110.9522705
10000.0000000     283.3379822    1455.3320313         0.0016624    1107.0883789
11000.0000000     281.3567810    1399.7375488         0.0016101    1103.2109375
12000.0000000     279.3756104    1345.8762207         0.0015591    1099.3200684
13000.0000000     277.3944092    1293.7042236         0.0015094    1095.4151611
14000.0000000     275.4132080    1243.1927393         0.0014609    1091.4963379
15000.0000000     273.4320068    1194.2706299         0.0014136    1087.5933545
16000.0000000     271.4508057    1146.9284653         0.0013675    1083.6163207
17000.0000000     269.4696045    1101.1173216         0.0013225    1079.6542738
18000.0000000     267.4884033    1056.7994385         0.0012787    1075.6782227
19000.0000000     265.5072021    1013.9373779         0.0012360    1071.6872559
20000.0000000     263.5261012     972.4935654         0.0011943    1067.6812334
```

The example below shows the running of 'ATMOS' when a single calculation is required:

```
.RU ATMOS
SET ATMOSPHERIC FLAG, KEYAIR (1,2,3,4,5 OR 6): 3
SINGLE CALCULATION, OR TABLE (1 OR 2): 1
STATE ALTITUDE (IN FEET): 15500
TEMPERATURE OF THE DAY, TDAY (IN DEG. C): 30
QNH OF THE DAY (IN MILLIBARS): 1013.25
HEIGHT OF THE AIRFIELD REFERENCE POINT, HAFR: 0
PAUSE
Type C to Continue, X to Exit, T to Trace.
*X

END OF EXECUTION
CPU TIME: 0.37  ELAPSED TIME: 1.04
EXIT

.

.TY ATMOS.OUT
 15500.0000000    272.4414063   1170.4053955      0.0013904   1085.5915527

.
```

APPENDIX 4 - PROGRAM 'ENERGY'

As mentioned in the introduction, program 'ENERGY' uses the same relatively simple energy balance equations as 'MCEP' to estimate the power components in steady level flight. However, the profile power equation used in 'MCEP' applies only to two-bladed rotors with zero twist. This equation has been modified to apply to helicopters having any number of rotor blades, but still with zero twist.

MCEP

$$\frac{C_{P_o}}{\sigma} = (1 + 4.6\mu^2)\left[\frac{\delta_o}{8} + \delta_1 \frac{7}{8a}\left(\frac{C_T}{\sigma}\right) + \delta_2 \frac{49}{8a^2}\left(\frac{C_T}{\sigma}\right)^2\right]$$

ENERGY

$$\frac{C_{P_o}}{\sigma} = (1 + 4.6\mu^2)\left[\frac{\delta_o}{8} + \delta_1 \frac{3}{4aB^3}\left(\frac{C_T}{\sigma}\right) + \delta_2 \frac{9}{2a^2B^6}\left(\frac{C_T}{\sigma}\right)^2\right]$$

where $C_{P_o} = \frac{P_o}{\rho\pi R^2(\Omega R)^3}$ and $C_T = \frac{T}{\rho\pi R^2(\Omega R)^2}$

It can be seen that the above equations are identical for B = 0.95, which is a typical value for a two-bladed rotor (Ref. 13). 'ENERGY' calculates tip loss factor 'B' using the standard equation:

$$B = 1 - \frac{\sqrt{2C_T}}{b}$$

In addition to the 'MCEP' relationships, equation 24 was incorporated into 'ENERGY' to enable estimation of instantaneous rate of climb for a given shaft power. Capability to estimate range and endurance has also been included using the Breguet equations. (Equations 25 and 26).

When a speed-power polar is required 'ENERGY' outputs a file 'ENRG.DAT' which is compatible with 'TRANS' (Ref. 11). If range, endurance or one specified velocity is required 'ENERGY' outputs a file 'ENERGY.OUT'. 'ENERGY' also has a file renaming feature similar to 'POLAR' (Appendix 1). Here however, files are renamed 'ENERG1.OUT' or 'ENRG1.DAT'. Up to five consecutive runs may be made without overwriting previous files. (as with 'POLAR')

A4.1 Loading 'ENERGY' on the DEC-10

For ARL users, a description is shown below of how to load program 'ENERGY' on the DEC-10 computer. 'ENERGY' is also available on the VAX-11 computer and is loaded in a similar fashion.

```
.PLE MX D/039
MESSAGE TRANSMITTED at 14:19


;;OPR: - D/039 MX DTA1
.AS DTA1
DTA001 assigned

.DIR DTA1:

Tape ID: D/039
Free: 428 blks, 13 files
POLAR .CCL     1   20-Apr-83
POLAR .FOR    60    7-Sep-83
DING  .REL     1   14-Apr-83
ENERGY.CCL     1   20-Apr-83
ENERGY.FOR    32    7-Sep-83
BLADE .FOR    48    6-Jun-83
TSUB81.REL    10   30-Mar-83
CPU   .REL   [illegible]   [illegible]-Nov-7[illegible]
ATMOS .FOR    15   23-Jun-83

.COPY/X=DTA1:ENERGY.*,TSUB81.*,CPU.*,DING.*

.COM ENERGY
FORTRAN: ENERGY
ENERGY
CONTRL
NEW
POW
CLIMB
ENDRNG

.TY ENERGY.CCL
ENERGY,TSUB81,CPU,DING
ENERGY/SAV=
/GO

.R LINK

*@ENERGY

EXIT

.DIR ENERGY.EXE

ENERGY  EXE    44  <057>   7-Sep-83   DSKD:  [1031,1063]

.
```

A4.2 Speed-Power Polar

When a speed-power polar is required, a file 'ENERGY.DAT' is created by 'ENERGY' which is suitable for use with 'TRANS'.

An example of running 'ENERGY' is demonstrated below, together with the subsequent running of 'TRANS'.

For this case, the drag divergent Mach Number is assumed to be 0.75, and the thrust divergence coefficients are 0.1 and 0.2. (TC1 and TC2 respectively). Definitions of induced velocity factor, drag divergent Mach Number, TC1 and TC2 may be found in 'MCEP' documentation. [Refs. 3 and 4].

```
.RU ENERGY


TITLE (TWO LINES OF UP TO 60 CHARACTERS)
: ENERGY METHOD (MCEP)
SPEED POWER POLAR

ARE UNITS IN IMPERIAL OR METRIC (I OR M) ? M

ALL UP WEIGHT (N OR LB) = ? 42718

ROTOR TIP SPEED (M/S OR FT/S) = ? 218.6

ROTOR RADIUS (M OR FT) = ? 6.4

ROTOR BLADE CHORD (M OR FT) = ? 0.395

NUMBER OF ROTOR BLADES = ? 4

2D LIFT CURVE SLOPE (/RAD) = ? 5.73

INDUCED VELOCITY FACTOR = ? 2.14

SPEED OF SOUND (M/S OR FT/S) = ? 349

EQUIVALENT FLAT PLATE AREA (M**2 OR FT**2) = ? 1.58

DRAG DIVERGENT MACH NUMBER = ? 0.75

DRAG POLAR CO-EFFICIENT (DEL0) = ? 0.0087

DELTA1 (/RAD) = ? -0.0216

DELTA2 (/RAD**2) = ? 0.4

AIR DENSITY (KG/M**3 OR SLUG/FT**3) = ? 1.1644

THRUST DIVERGENCE CO-EFFICIENTS (TC1) = ? 0.1

TC2 = ? 0.2

IS RANGE/ENDURANCE REQUIRED ? (Y OR N) : N

IS A SPEED-POWER POLAR REQUIRED ? (Y OR N) : Y

MAXIMUM VELOCITY (KNOTS) = ? 150

VELOCITY INTERVAL (KNOTS) = ? 2


END OF EXECUTION
CPU TIME: 1.78  ELAPSED TIME: 1:6.76
EXIT

.
```

'TRANS' has a plotting facility whereby a previously created file 'POLAR.DAT' (using 'POLAR') can be used with the resulting 'ENRG.DAT' file to produce an overlay plot. This plot (Fig. 4) compares speed-power polars obtained using the two methods.

```
.RU PUB:TRANS

[TRANS version date 12-NOV-80]
I/P FILENAME = POLAR
    BLADE ELEMENT METHOD
     POWER - SPEED POLAR
I/P FILE RECORDED ON 27-Oct-83 AT 17:11
INTEGN INT = 0.0000E+00; RUN CPU TIME =  4.02 SEC.
TIME FROM 0.0000E+00 TO 1.5000E+02 IN STEPS OF 2.0000E+00
*SPA

IS SPACING BETWEEN PLOTS REQRD : N
*SCA
BLK NO. -1 DENOTES INDEP VARIABLE
ARE PLOT SCALE LIMITS TO BE READ FROM DSK : N
IS TTY LISTING OF LIMITS REQRD : N
ARE MODIFICATIONS REQRD : Y
BLK, LOWER, UPPER
1,0,1500

*PLS
STRIP PLOTS :
BLKS
1

TO SPECIFY NO. OF X UNITS/INCH, TYPE 0 FOR X
LENGTH OF AXES IN INCHES; X, Y = 0,6
MIN X, NO. OF X UNITS/INCH = 0,30
ARE SYMBOLS REQRD FOR PLOTS : N
LINE KEY (0 GIVES DEFAULT) = 0
*GOE
** RUNNING **

*REP
I/P FILENAME = ENRGY
ARE SYMBOLS REQRD FOR PLOTS : N
LINE KEY (0 GIVES DEFAULT) = 0
** RUNNING **

I/P FILENAME =

END OF EXECUTION
CPU TIME: 5.72  ELAPSED TIME: 2:26.10
EXIT
```

A4.3 Running 'ENERGY' for one particular airspeed

If desired, 'ENERGY' may be run for one particular airspeed, as shown below. The results apply to level flight, the climb rate being the instantaneous climb rate possible appropriate to excess power, not the sustained climb rate. The results for this operation are output on to file 'ENERGY.OUT'.

```
.RU ENERGY

TITLE (TWO LINES OF UP TO 60 CHARACTERS)
: ENERGY METHOD (MCEP)
RATE OF CLIMB

ARE UNITS IN IMPERIAL OR METRIC (I OR M) ? M

ALL UP WEIGHT (N OR LB) = ? 42718

ROTOR TIP SPEED (M/S OR FT/S) = ? 218.6

ROTOR RADIUS (M OR FT) = ? 6.4

ROTOR BLADE CHORD (M OR FT) = ? 0.395

NUMBER OF ROTOR BLADES = ? 4

2D LIFT CURVE SLOPE (/RAD) = ? 5.73

INDUCED VELOCITY FACTOR = ? 2.14

SPEED OF SOUND (M/S OR FT/S) = ? 349

EQUIVALENT FLAT PLATE AREA (M**2 OR FT**2) = ? 1.58

DRAG DIVERGENT MACH NUMBER = ? 0.75

DRAG POLAR CO-EFFICIENT (DEL0) = ? 0.0087

DELTA1 (/RAD) = ? -0.0216

DELTA2 (/RAD**2) = ? 0.4

AIR DENSITY (KG/M**3 OR SLUG/FT**3) = ? 1.1644

THRUST DIVERGENCE CO-EFFICIENTS (TC1) = ? 0.1

TC2 = ? 0.2

IS RANGE/ENDURANCE REQUIRED ? (Y OR N) : N

IS A SPEED-POWER POLAR REQUIRED ? (Y OR N) : N

VELOCITY IN KNOTS (0 KNOTS IS MINIMUM ALLOWABLE) ? : 50

WHAT IS POWER AVAILABLE AT MAIN ROTOR SHAFT (HP) ? : 1275

STOP

END OF EXECUTION
CPU TIME: 1.17  ELAPSED TIME: 42.26
EXIT

.
```

```
 TY ENERGY.OUT
ENERGY METHOD (MCEP)
RATE OF CLIMB
 ALL UP WEIGHT =    42718.0000000 N
 AIR DENSITY =        1.1644000 KG/M**3
 TIP SPEED =     218.6000000 M/S
 ROTOR RADIUS =       6.4000000 M
 ROTOR CHORD =        0.3950000 M
 NUMBER OF ROTOR BLADES =        4.0000000
 LIFT CURVE SLOPE =       5.7300000 /RAD
 BLADE SOLIDITY =       0.0785828
 EQUIVALENT FLAT PLATE AREA =       1.5800000 M**2
 INDUCED VELOCITY FACTOR =       2.1400000
 DRAG POLAR CO-EFFICIENT(DELTA0)=        0.0087000
 DELTA 1 =       -0.0216000 /RAD
 DELTA 2 =        0.4000000 /RAD**2

 ADVANCE RATIO(MU) =        0.1176670
 FLAT PLATE DRAG =      608.6095000 N
 THRUST =    42722.3300000 N
 DISC AREA =      128.6796000 M**2
 SPEED OF SOUND =      349.0000000 M/S
 DRAG DIVERGENT MACH NO. =        0.7500000
 TC1 =        0.1000000
 TC2 =        0.2000000
 INDUCED VELOCITY =        5.4247720 M/S
 COMPRESSIBILITY POWER =        0.0000000 HP
 INDUCED POWER =      325.6911000 HP
 PARASITE POWER =       20.9960500 HP
 PROFILE POWER =      215.3766000 HP
 TOTAL POWER =      562.0638000 HP
 AVAILABLE SHAFT POWER =     1275.0000000 HP
 CLIMB POWER =      712.9362000 HP
 RATE OF CLIMB =      746.6153000 M/MIN
 STALL POWER =        0.0000000 HP
 AIRSPEED =       50.0000000 KT
```

A4.4 Estimating Range and Endurance using 'ENERGY'

To estimate the range and endurance for a given helicopter, 'ENERGY' is run as in the previous example, with the following changes:

```
IS RANGE/ENDURANCE REQUIRED ? (Y OR N) : Y

VELOCITY INTERVAL (KNOTS) = ? 2

SPECIFIC FUEL CONSUMPTION ? (KG/HP-HR OR LB/HP-HR) : 0.504

WEIGHT OF FUEL CARRIED ? (N OR LB) : 6400

STOP

END OF EXECUTION
CPU TIME: 1.35  ELAPSED TIME: 48.02
EXIT
```

The output file is shown below:

```
.TY ENERGY.OUT
ENERGY METHOD (MCEP)
RANGE/ENDURANCE
 ALL UP WEIGHT =    42718.0000000 N
 AIR DENSITY =         1.1644000 KG/M**3
 TIP SPEED =       218.6000000 M/S
 ROTOR RADIUS =         6.4000000 M
 ROTOR CHORD =         0.3950000 M
 NUMBER OF ROTOR BLADES =         4.0000000
 LIFT CURVE SLOPE =         5.7300000 /RAD
 BLADE SOLIDITY =         0.0785828
 EQUIVALENT FLAT PLATE AREA =         1.5800000 M**2
 INDUCED VELOCITY FACTOR =         2.1400000
 DRAG POLAR CO-EFFICIENT(DELTA0)=         0.0087000
 DELTA 1 =         -0.0216000 /RAD
 DELTA 2 =          0.4000000 /RAD**2
 S.F.C. =  0.504 KG/HP-HR
 FUEL WEIGHT =  6400.0 N

 RESULTS FOR ENDURANCE OF  2.75 HOURS, ARE AS FOLLOWS :

 ADVANCE RATIO(MU) =         0.1647338
 FLAT PLATE DRAG =      1192.8750000 N
 THRUST =   42734.6500000 N
 DISC AREA =       128.6796000 M**2
 SPEED OF SOUND =       349.0000000 M/S
 DRAG DIVERGENT MACH NO. =         0.7500000
 TC1 =         0.1000000
 TC2 =         0.2000000
 INDUCED VELOCITY =         3.9283030 M/S
 COMPRESSIBILITY POWER =         0.0000000 HP
 INDUCED POWER =       225.1538000 HP
 PARASITE POWER =        57.6131700 HP
 PROFILE POWER =       227.7853000 HP
 TOTAL POWER =       510.5522000 HP
 STALL POWER =         0.0000000 HP
 AIRSPEED =        70.0000000 KT
1 RESULTS FOR RANGE OF     440.8 KM, ARE AS FOLLOWS :

 ADVANCE RATIO(MU) =         0.2494540
 FLAT PLATE DRAG =      2735.3350000 N
 THRUST =   42805.4900000 N
 DISC AREA =       128.6796000 M**2
 SPEED OF SOUND =       349.0000000 M/S
 DRAG DIVERGENT MACH NO. =         0.7500000
 TC1 =         0.1000000
 TC2 =         0.2000000
 INDUCED VELOCITY =         2.6054610 M/S
 COMPRESSIBILITY POWER =        16.1808900 HP
 INDUCED POWER =       149.5816000 HP
 PARASITE POWER =       200.0531000 HP
 PROFILE POWER =       260.6611000 HP
 TOTAL POWER =       626.4767000 HP
 STALL POWER =         0.0000000 HP
 AIRSPEED =       106.0000000 KT
```

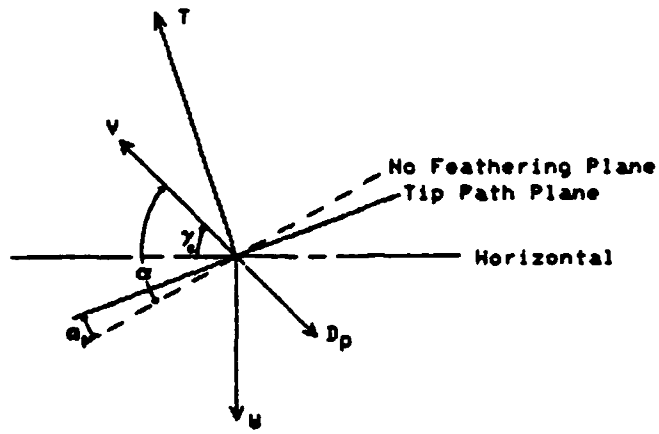

Figure 1 : Forces acting on helicopter in steady flight

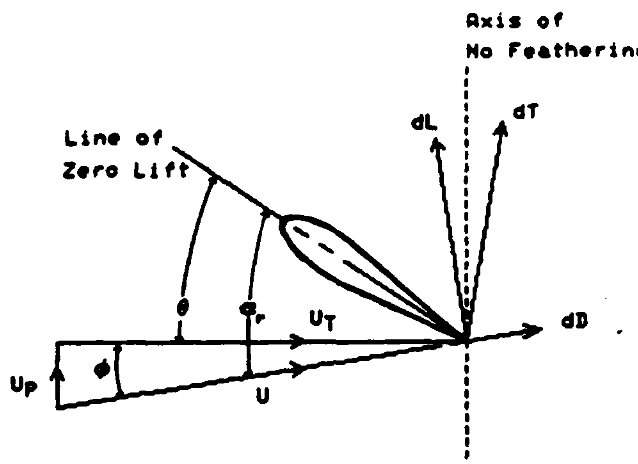

Figure 2 : Blade Element Representation

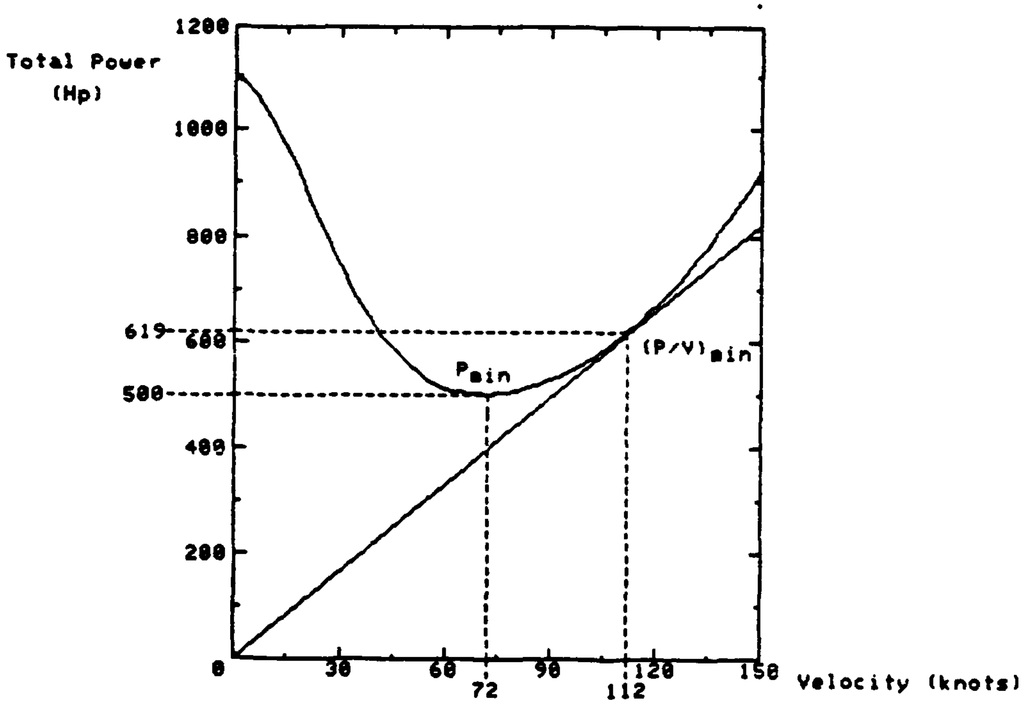

Figure 3 : Speed - Power Polar

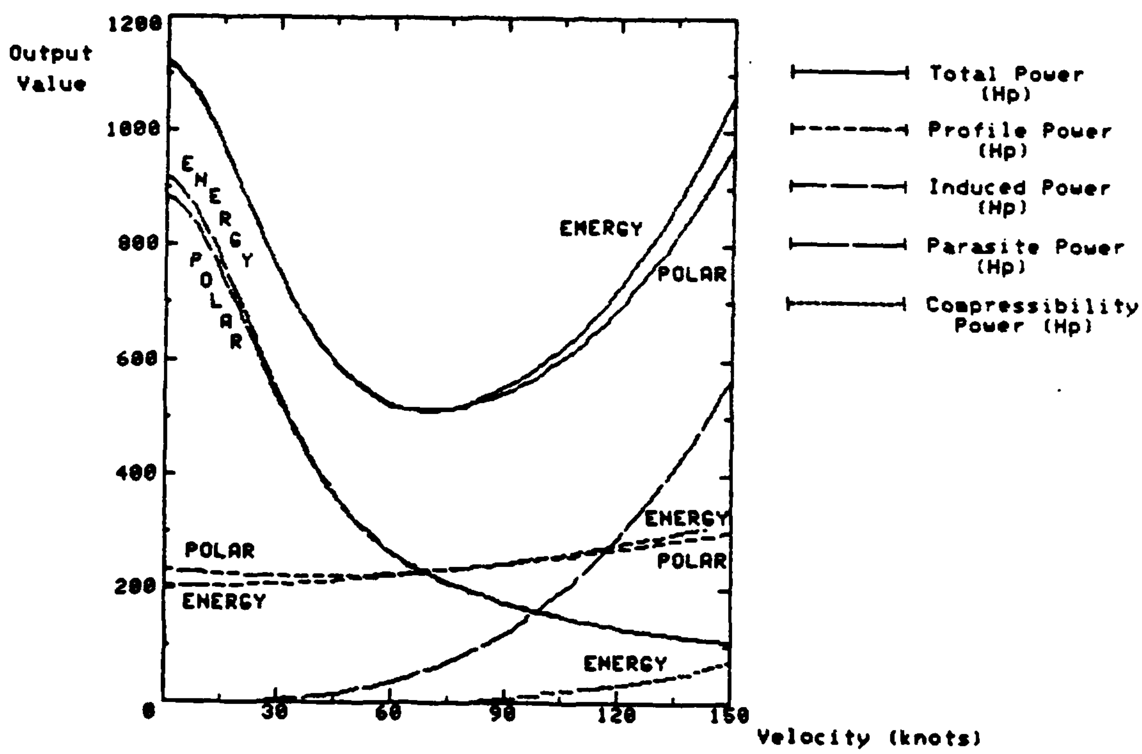

Figure 4 : Comparison Of 'POLAR' And 'ENERGY' For Helicopter Having Main Rotor Blades With Zero Degrees Twist

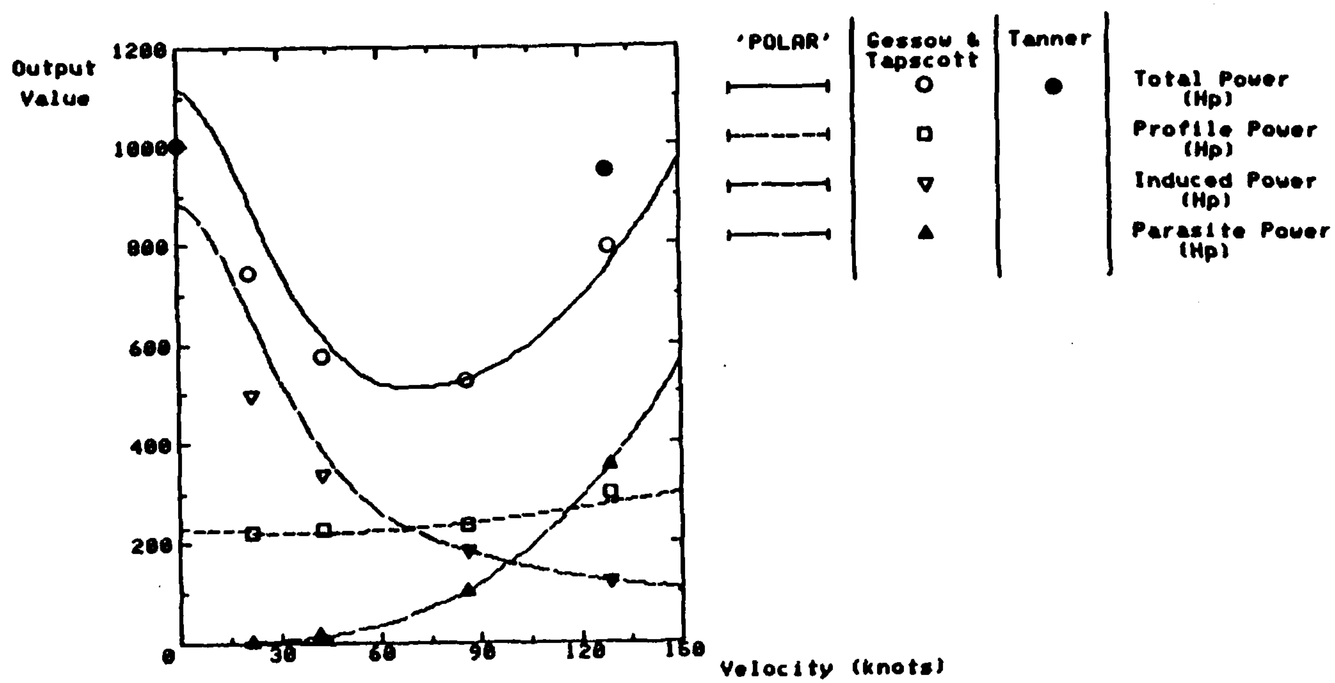

Figure 5 : Comparison Of 'POLAR', Gessow & Tapscott And Tanner For A Main Rotor Having Zero Degrees Twist

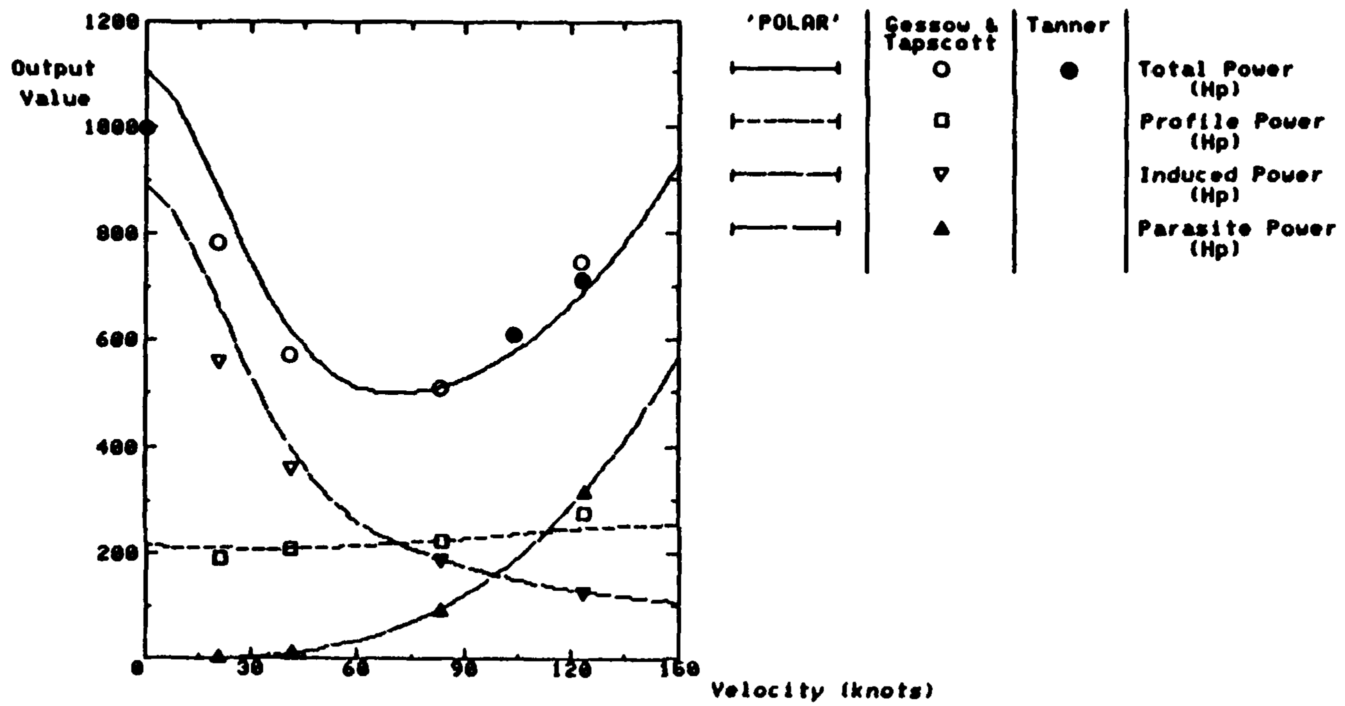

Figure 6 : Comparison Of 'POLAR', Gessow & Tapscott And Tanner For A Main Rotor Having Eight Degrees Twist

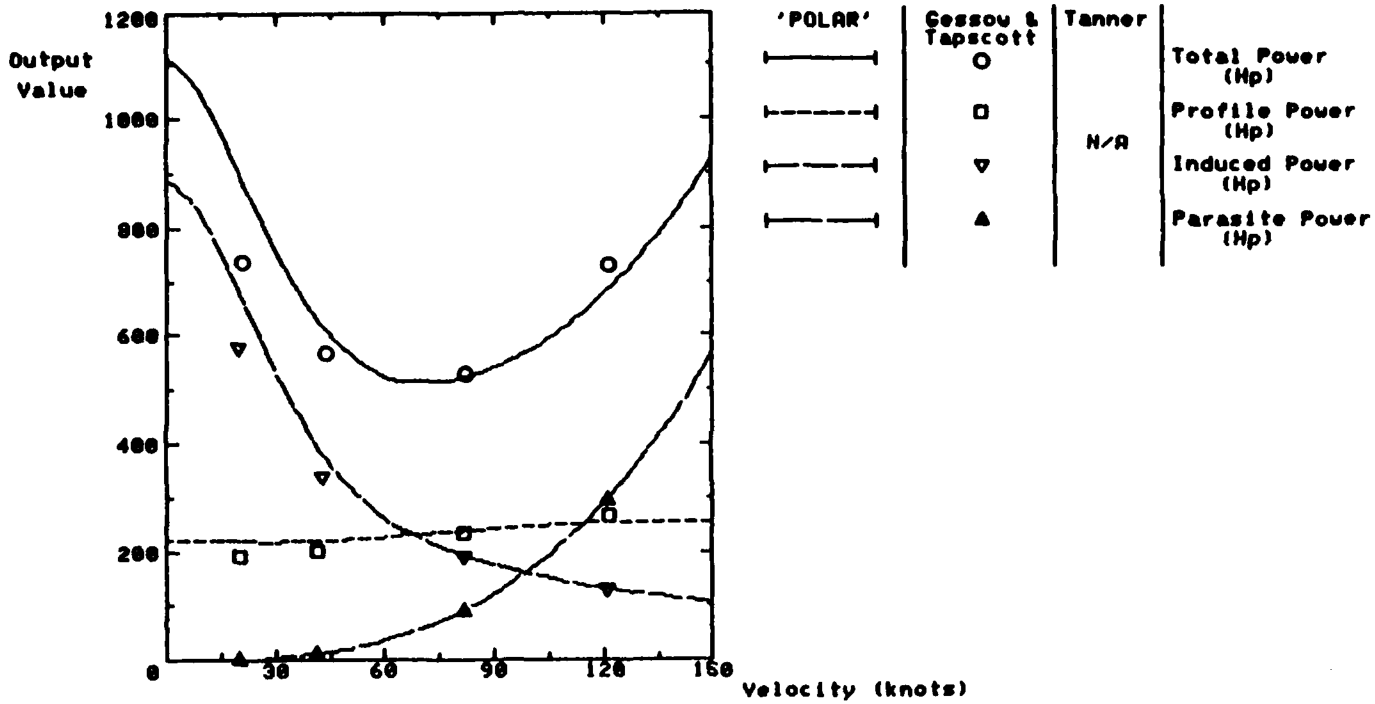

Figure 7 : Comparison Of 'POLAR', Gessow & Tapscott And Tanner
For A Main Rotor Having Sixteen Degrees Twist

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ESTIMATION OF HELICOPTER PERFORMANCE USING A PROGRAM BASED ON BLADE ELEMENT ANALYSIS	a. document: UNCLASSIFIED b. title: U c. abstract: U	7. No Refs: 13

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16. Abstract

A convenient method of predicting helicopter performance is presented, which is applicable up to speeds corresponding to an advance ratio of 0.3, for any conventional helicopter (ie. single main rotor) with flapping blades. This method uses a computer program, 'POLAR', which is based on a blade-element analysis assuming uniform induced flow. Program 'POLAR' can be employed for most steady flight conditions and is not subject to limitations imposed by the use of performance tables and charts. Details of blade operating conditions may be estimated at specified points on the rotor disc. The structure of the program and examples of its use are given. Comparisons of estimates obtained using 'POLAR', with other performance methods are also included.

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16. Abstract (Contd)		
17. Imprint Aeronautical Research Laboratories, Melbourne.		
18. Document Series and Number Aerodynamics Technical Memorandum 365	19. Cost Code 512102	20. Type of Report and Period Covered
21. Computer Programs Used		
22. Establishment File Ref(s)		

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